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Calculating three loop ladder and V -topologies for massive operator matrix elements by computer algebra. (English) Zbl 1348.81034
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Summary: Three loop ladder and V -topology diagrams contributing to the massive operator matrix element A_{Qg} are calculated. The corresponding objects can all be expressed in terms of nested sums and recurrences depending on the Mellin variable N and the dimensional parameter ε . Given these representations, the desired Laurent series expansions in ε can be obtained with the help of our computer algebra toolbox. Here we rely on generalized hypergeometric functions and Mellin-Barnes representations, on difference ring algorithms for symbolic summation, on an optimized version of the multivariate Almkvist-Zeilberger algorithm for symbolic integration, and on new methods to calculate Laurent series solutions of coupled systems of differential equations. The solutions can be computed for general coefficient matrices directly for any basis also performing the expansion in the dimensional parameter in case it is expressible in terms of indefinite nested product-sum expressions. This structural result is based on new results of our difference ring theory. In the cases discussed we deal with iterative sum- and integral-solutions over general alphabets. The final results are expressed in terms of special sums, forming quasi-shuffle algebras, such as nested harmonic sums, generalized harmonic sums, and nested binomially weighted (cyclotomic) sums. Analytic continuations to complex values of N are possible through the recursion relations obeyed by these quantities and their analytic asymptotic expansions. The latter lead to a host of new constants beyond the multiple zeta values, the infinite generalized harmonic and cyclotomic sums in the case of V -topologies.

MSC:

- 81-08 Computational methods for problems pertaining to quantum theory
- 81T80 Simulation and numerical modelling (quantum field theory) (MSC2010)
- 81V05 Strong interaction, including quantum chromodynamics
- 81U35 Inelastic and multichannel quantum scattering

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Keywords:

massive 3-loop integrals in QCD; master integrals; automation and method of differential equations; almkvist-Zeilberger algorithm; Mellin-Barnes method

Software:

[EvaluateMultiSums](#); [Fermat](#); [HarmonicSums](#)

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