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**An adjoint view on flux consistency and strong wall boundary conditions to the Navier-Stokes equations.** (English) Zbl 1349.76390

J. Comput. Phys. 301, 247-264 (2015).

Summary: Inconsistent discrete expressions in the boundary treatment of Navier-Stokes solvers and in the definition of force objective functionals can lead to discrete-adjoint boundary treatments that are not a valid representation of the boundary conditions to the corresponding adjoint partial differential equations. The underlying problem is studied for an elementary 1D advection-diffusion problem first using a node-centred finite-volume discretisation. The defect of the boundary operators in the inconsistently defined discrete-adjoint problem leads to oscillations and becomes evident with the additional insight of the continuous-adjoint approach. A homogenisation of the discretisations for the primal boundary treatment and the force objective functional yields second-order functional accuracy and eliminates the defect in the discrete-adjoint boundary treatment. Subsequently, the issue is studied for aerodynamic Reynolds-averaged Navier-Stokes problems in conjunction with a standard finite-volume discretisation on median-dual grids and a strong implementation of noslip walls, found in many unstructured general-purpose flow solvers. Going out from a base-line discretisation of force objective functionals which is independent of the boundary treatment in the flow solver, two improved flux-consistent schemes are presented; based on either body wall-defined or farfield-defined control-volumes they resolve the dual inconsistency. The behaviour of the schemes is investigated on a sequence of grids in 2D and 3D.

**MSC:**

76M12 Finite volume methods applied to problems in fluid mechanics
65N08 Finite volume methods for boundary value problems involving PDEs
76D05 Navier-Stokes equations for incompressible viscous fluids
35Q30 Navier-Stokes equations

Cited in **2** Documents

**Keywords:**

adjoint Navier-Stokes; dual consistency; strong boundary treatment

**Software:**

TAF; TAU

**Full Text:** DOI

**References:**

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