

**Yao, Weigang; Liou, Meng-Sing**

**A nonlinear modeling approach using weighted piecewise series and its applications to predict unsteady flows.** (English) [Zbl 1349.76149](#)

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Summary: To preserve nonlinearity of a full-order system over a range of parameters of interest, we propose an accurate and robust nonlinear modeling approach by assembling a set of piecewise linear local solutions expanded about some sampling states. The work by Rewiński and White [1] on micromachined devices inspired our use of piecewise linear local solutions to study nonlinear unsteady aerodynamics. These local approximations are assembled via nonlinear weights of radial basis functions. The efficacy of the proposed procedure is validated for a two-dimensional airfoil moving with different pitching motions, specifically AGARD's CT2 and CT5 problems [27], in which the flows exhibit different nonlinear behaviors. Furthermore, application of the developed aerodynamic model to a two-dimensional aero-elastic system proves the approach is capable of predicting limit cycle oscillations (LCOs) by using AGARD's CT6 [28] as a benchmark test. All results, based on inviscid solutions, confirm that our nonlinear model is stable and accurate, against the full model solutions and measurements, and for predicting not only aerodynamic forces but also detailed flowfields. Moreover, the model is robust for inputs that considerably depart from the base trajectory in form and magnitude. This modeling provides a very efficient way for predicting unsteady flowfields with varying parameters because it needs only a tiny fraction of the cost of a full-order modeling for each new condition—the more cases studied, the more savings rendered. Hence, the present approach is especially useful for parametric studies, such as in the case of design optimization and exploration of flow phenomena.

**MSC:**

[76G25](#) General aerodynamics and subsonic flows

**Keywords:**

modeling of nonlinear dynamics with parametric inputs; weighted piecewise linear (WPL) model; unsteady flow; limited cycle oscillations

**Software:**

[AUSM](#); [TAPENADE](#)

**Full Text:** [DOI](#)

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