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The Riemann constant for a non-symmetric Weierstrass semigroup. (English) Zbl 1360.14090

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The Riemann constant is an invariant that is associated to a pointed curve (X, P), whose Abel map is normalized at P. In this paper, the authors work over the complex numbers, assume that X is a compact Riemann surface (a curve) of genus $g > 1$ and use standard convention. They state the algebraic-transcendental correspondence for the Riemann constant on a general pointed curve and its consequences for the Jacobi inversion problem. The zero divisor of the theta function of a compact Riemann surface X of genus g is the canonical theta divisor of $\mathrm{Pic}^{(g-1)}$ up to translation by the Riemann constant Δ for a base point P of X. The complement of the Weierstrass gaps at the base point P gives a numerical semigroup, called the Weierstrass semigroup. It is classically known that the Riemann constant Δ is a half period, namely an element of $\frac{1}{2}\Gamma_\tau$, for the Jacobi variety $J(X) = \mathbb{C}^g/\Gamma_\tau$ of X if and only if the Weierstrass semigroup at P is symmetric. The aim of this paper is to analyze the non-symmetric case. Using a semi-canonical divisor D_0, the authors express the relation between the Riemann constant Δ and a half period in the non-symmetric case. They point out an application to an algebraic expression for the Jacobi inversion problem. They also identify the semi-canonical divisor D_0 for trigonal pointed curves, namely with total ramification at P.

Reviewer: Ahmed Lesfari (El Jadida)

MSC:

14H55 Riemann surfaces; Weierstrass points; gap sequences
14H50 Plane and space curves
14K25 Theta functions and abelian varieties
14H40 Jacobians, Prym varieties

Keywords:

Riemann constant; non-symmetric Weierstrass semigroup; theta function; Abel map; sigma function

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References:

[1] J.D. Fay, Theta functions on Riemann Surfaces, Lecture Notes in Mathematics, Vol. 352, Springer-Verlag, Berlin-New York, 1973. · Zbl 0281.30013

[2] J. Herzog, Generators and relations of Abelian semigroup and semigroup ring, Manuscripta Math. \textbf{3} (1970), 175-193 · Zbl 0211.33801

[3] Lewittes, J., Riemann surfaces and the theta functions, Acta Math., 111, 37-61, (1964) · Zbl 0125.31803

[4] H. M. Farkas and S. Zemel, Generalizations of Thomae's Formula for Zn Curves (Developments in Mathematics), Springer, New York, 2010. · Zbl 1222.14001

[5] J. Komeda, S. Matsutani, and E. Previato, The sigma function for Weierstrass semigroup ${⟨3,7,8⟩}$ and ${⟨6,13,14,15,16⟩}$, Int. J. Math \textbf{24} (2013), 1350085 (58 pages). · Zbl 1284.14045

[6] S. Matsutani and J. Komeda, Sigma functions for a space curve of type (3, 4, 5), J. Geom. Symm. Phys. \textbf{30} (2013), 75-91. · Zbl 1311.14033

[7] D. Mumford, Tata Lectures on Theta I, Progress in Mathematics, 28, Birkhäuser, Boston, 1983. · Zbl 0509.14049

[8] D. Mumford, Tata Lectures on Theta II, Progress in Mathematics, 43, Birkhäuser, Boston, 1984. · Zbl 0549.14014

[9] Pinkham, H.C., Deformation of algebraic varieties with G_{}\{m\} action, Astérisque, 20, 1-131, (1974) · Zbl 0304.14006

[10] Shiga, H., On the representation of the Picard modular function by ${θ}$ constants I-II, Publ. RIMS, Kyoto Univ., 24, 311-360, (1988) · Zbl 0678.10020

[11] Stöhr, K.-O., On the moduli spaces of Gorenstein curves with symmetric Weierstrass semigroups, J. Reine Angew. Math., 441, 189-213, (1993) · Zbl 0771.14009

paper as accurately as possible without claiming the completeness or perfect precision of the matching.