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Dzhafarov, Damir D.; **Patey, Ludovic**; **Solomon, Reed**; **Westrick, Linda Brown**

Ramsey's theorem for singletons and strong computable reducibility. (English) Zbl 1423.03159

Proc. Am. Math. Soc. 145, No. 3, 1343-1355 (2017).

Summary: We answer a question posed by *D. R. Hirschfeldt* and *C. G. Jockusch jun.* [J. Math. Log. 16, No. 1, Article ID 1650002, 59 p. (2016; Zbl 1373.03068)] by showing that whenever $k > \ell$, Ramsey's theorem for singletons and k-colorings, RT^1_k, is not strongly computably reducible to the stable Ramsey's theorem for ℓ-colorings, SRT^2_ℓ. Our proof actually establishes the following considerably stronger fact: given $k > \ell$, there is a coloring $c : \omega \to k$ such that for every stable coloring $d : [\omega]^2 \to \ell$ (computable from c or not), there is an infinite homogeneous set H for d that computes no infinite homogeneous set for c. This also answers a separate question of the first author [J. Symb. Log. 81, No. 4, 1405–1431 (2016; Zbl 1368.03044)], as it follows that the cohesive principle, COH, is not strongly computably reducible to the stable Ramsey's theorem for all colorings, $\mathsf{SRT}^2_{<\infty}$. The latter is the strongest partial result to date in the direction of giving a negative answer to the longstanding open question of whether COH is implied by the stable Ramsey's theorem in ω-models of RCA_0.

MSC:

03D80 Applications of computability and recursion theory
03F35 Second- and higher-order arithmetic and fragments
05D10 Ramsey theory
03B30 Foundations of classical theories (including reverse mathematics)
03D30 Other degrees and reducibilities in computability and recursion theory

Full Text: DOI arXiv

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