Grayson, Matthew A.

The heat equation shrinks embedded plane curves to round points. (English) Zbl 0667.53001

J. Differ. Geom. 26, 285-314 (1987).

This paper contains the final solution of the long-standing "curve- shortening problem" for plane curves: Let $\gamma_0 : S^1 \to \mathbb{R}^2$ be a regular embedded closed plane curve. Then the evolution equation $\dot{\gamma} = k \cdot N$ (N a unit normal field, k the curvature) with initial condition $\gamma(0, s) = \gamma_0(s)$ always has a solution $\gamma : \mathbb{R}^+ \times S^1 \to \mathbb{R}^2$, $S \mapsto \gamma_t(s) = \gamma(t, s)$ is an embedded curve for all t and γ_t approaches a (shrinking) round circle as $t \to \infty$.

Reviewer: U.Pinkall

MSC:

53A04	Curves in Euclidean and related spaces
35G10	Initial value problems for linear higher-order PDEs
35K25	Higher-order parabolic equations

Keywords:

curve-shortening problem; closed plane curve; evolution equation

Full Text: DOI