

**Atiyah, Michael**

**New invariants of 3- and 4-dimensional manifolds.** (English) Zbl 0667.57018

The mathematical heritage of Hermann Weyl, Proc. Symp., Durham/NC 1987, Proc. Symp. Pure Math. 48, 285-299 (1988).

[For the entire collection see [Zbl 0644.00001](#).]

The Casson invariant  $\lambda(Y)$  of an oriented homology 3-sphere  $Y$  is roughly defined by  $2\lambda(Y) = \{\text{number of irreducible representations } \pi_1(Y) \rightarrow SU(2)\}$ . It can be viewed as the Euler characteristic  $\sum_{q=0}^7 (-1)^q \dim HF_q^+(Y)$  of the Witten-Morse complex  $C_*$  for the Chern-Simons function  $f: \mathcal{C} \rightarrow R/Z$ , where the infinite-dimensional manifold  $\mathcal{C}$  is the space of (classes of)  $SU(2)$ -connections for the trivial bundle over  $Y$ . HF stands for Floer homology. There is a deep connection between the Donaldson invariants  $\Phi_k$  of an indecomposable 4-manifold  $W$  having splittable intersection form, and Floer homology of a homology 3-sphere  $Y \subset W$  which realizes the splitting. Not all of the above results are fully written down, but the general picture seems to be fairly clear and the ideas are so beautiful and simple that they deserve a nontechnical presentation. The author fulfils such a presentation in a brilliant manner.

Reviewer: [S.V.Matveev](#)

**MSC:**

- [57R55](#) Differentiable structures in differential topology
- [57N13](#) Topology of the Euclidean 4-space, 4-manifolds (MSC2010)
- [81T17](#) Renormalization group methods applied to problems in quantum field theory
- [58E05](#) Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
- [37J99](#) Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
- [57N10](#) Topology of general 3-manifolds (MSC2010)

Cited in **7** Reviews  
Cited in **15** Documents

**Keywords:**

Witten-Morse complex for the Chern-Simons function; Donaldson invariants of an indecomposable 4-manifold; Casson invariant;  $SU(2)$ -connections; Floer homology