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Complex convexity of Orlicz modular sequence spaces. (English) Zbl 1383.46009

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E. Thorp and *R. Whitley* [Proc. Am. Math. Soc. 18, 640–646 (1967; Zbl 0185.20102)] showed that the strong maximum modulus theorem for analytic functions with values in a complex Banach space $(X, \|\cdot\|_X)$ holds, provided that each point of the unit sphere of X is a complex extreme point. Thereafter, complex strict convexity, complex uniform convexity and complex midpoint local uniform convexity have been studied intensively in (quasi-)Banach spaces and (quasi-)Banach lattices in different directions. The authors of the present paper consider the notion of complex strict convexity, complex midpoint local uniform convexity (and the appropriate points for the local approach) in the modular sense. It means that in the respective definitions (of complex (strongly) extreme point) we deal with general modular spaces X_ρ replacing the norm $\|\cdot\|_X$ by the modular ρ and taking the element x from the modular sphere $S(X_\rho)$ (Definition 2 and 3). Denote the respective properties with the letter ρ at the beginning. The authors characterize ρ-complex strongly extreme points by a suitable modulus introduced in Definition 7 (see Proposition 8). They also prove that, if a modular space X_ρ is ρ-complex midpoint locally uniformly convex, then it is ρ-complex strictly convex. Finally, the last theorem in the paper states that each Orlicz modular sequence space $l_{\Phi,\rho}$ is ρ-complex midpoint locally uniformly convex (Theorem 15).

The questions about the relations between the complex strict convexity and ρ-complex strict convexity seem to be natural and interesting (similarly for points). However, the authors do not discuss them.

Reviewer: Pawel Kolwicz (Poznań)

MSC:

46A80 Modular spaces
46B20 Geometry and structure of normed linear spaces
46A45 Sequence spaces (including Köthe sequence spaces)

Keywords:

complex extreme point; complex strongly extreme point; modular; modular space; Orlicz sequence space

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