Mirmostafaee, Alireza Kamel; **Piotrowski, Zbigniew**
On the preservation of Baire and weakly Baire category. (English) Zbl 1413.54091
Math. Bohem. 141, No. 4, 475-481 (2016).

Recall that a topological space is said to be a *Baire space* if no non-empty open subset is of first category. *G. Beer* and *L. Villar* [Southeast Asian Bull. Math. 11, No. 2, 127–133 (1988; Zbl 0665.54019)] extended this notion by introducing the class of weakly Baire spaces. By a *weakly Baire space*, we mean a space in which no non-empty open, dense-in-itself subset is countable. Clearly, every T_1 Baire space is weakly Baire.

In the paper under review, the authors study the preservation of Baire and weakly Baire spaces under images and/or preimages of special kinds of mappings. It is shown that a space Y is Baire provided that there exist a Baire space X and a surjection $f : X \to Y$ satisfying the following conditions: (1) $f(U)$ has non-empty interior in Y whenever U is a non-empty open subset of X; and (2) if U is a non-empty open subset of X and W is a non-empty open subset of Y such that $W \subseteq f(U)$, then there exists a non-empty open $U' \subseteq U$ such that $f(U') \subseteq W$. This slightly improves a result of *Z. Frolík* in [Czech. Math. J. 11(86), 381–385 (1961; Zbl 0104.17204)]. A similar result is obtained for weakly Baire spaces. Finally, a result on the preservation of Baireness under preimages along the framework of *D. Noll* in [Proc. Am. Math. Soc. 107, No. 3, 847–854 (1989; Zbl 0687.54012)] is also obtained.

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MSC:

54E52	Baire category, Baire spaces
54C08	Weak and generalized continuity
54C10	Special maps on topological spaces (open, closed, perfect, etc.)

Keywords:

Baire space; feebly continuous; feebly open; quasi-continuous; quasi-interior continuous; weakly Baire space; fiber-completeness

Full Text: DOI