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Topology of conic bundles. (English) Zbl 0668.14013

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Let $P \to X$ be a bundle of conics on a smooth algebraic variety X which degenerates into a pair of distinct lines over a smooth irreducible divisor Y. The 2-sheeted covering of Y thus obtained defines an element $\alpha \in H^1(Y, \mathbb{Z}/2)$. On the other hand we have a $\mathbb{P}^1$-bundle on X-Y, and the topological obstruction to this $\mathbb{P}^1$- bundle to be SL(2)-banal, that is, to be the projective bundle of a rank 2-topological vector bundle with trivial determinant, is an element $\beta \in H^2(X-Y, \mathbb{Z}/2)$ (see § 1.1). Consider the Gysin map $H^2(X-Y, \mathbb{Z}/2) \to H^1(Y, \mathbb{Z}/2)$, the composite of the coboundary map $H^2(X-Y, \mathbb{Z}/2) \to H^3(X, X-Y, \mathbb{Z}/2)$ with the Thom isomorphism $H^3(X, X-Y, \mathbb{Z}/2) \to H^1(Y, \mathbb{Z}/2)$, by definition.

Theorem 1. If the total space P of the conic bundle is a smooth algebraic variety then under the Gysin map, the image of the obstruction class $\beta \in H^2(X-Y, \mathbb{Z}/2)$ is the cohomology class $\alpha \in H^1(Y, \mathbb{Z}/2)$ defined by the 2-sheeted covering. In particular, if the 2-sheeted covering is not split, then the $\mathbb{P}$-bundle on X-Y is not topologically SL(2)-banal.

Corollary 1. Under the hypothesis of theorem 1, the topological Brauer class $\beta' \in H^3(X-Y, \mathbb{Z})$ of the $\mathbb{P}^1$-bundle (see § 1.1) maps under the Gysin homomorphism to the Chern class $\alpha' \in H^2(Y, \mathbb{Z})$ of the line bundle determined by the 2-sheeted cover of Y. In particular if the Chern class is not zero, then the $\mathbb{P}^1$-bundle on X-Y is not topologically banal, that is, it is not associated to any rank-2 topological vector bundle.

MSC:

14F45	Topological properties in algebraic geometry
14F05	Sheaves, derived categories of sheaves, etc. (MSC2010)
57R20	Characteristic classes and numbers in differential topology
55S35	Obstruction theory in algebraic topology

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