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Lyapunov stability for some central forces. (English) Zbl 0668.34051

J. Differ. Equations 74, No. 2, 254-265 (1988).

Consider the system $(*)$: $\ddot{x} + xf(x) = 0$, $\ddot{y} + yf(x) = 0$, where $f \in C^0(A, R)$, $0 \in A = \overset{\circ}{A} \subset \mathbb{R}$. If $f(0) > 0$ then the force is attractive and one could think that the stability always occurs as in the conservative case. This naive conjecture is false as proved by *A. Barone* and the author in the paper Attractive central forces may yield Lyapunov instability. Proceed. VII ELAM, Equinoccio, Caracas, 1986, 105-112. They find more than only an example for this conjecture, and obtain large classes of f for which instability occurs and they completely solve the stability problem for f even: the equilibrium is then stable only in the trivial case, i.e. whenever f is locally constant at zero and $f(0) > 0$. Is stability possible just in the trivial case? The answer to this question is negative. The aim of this paper is to determine and construct all the continuous maps f such that the origin is a stable equilibrium for the system $(*)$. This paper gives also a different proof of results on stability using an integral equation similar to Abel's equation, and introduce some results on the unstable cases.

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