Komeda, Jiryo; Takahashi, Takeshi

Relating Galois points to weak Galois Weierstrass points through double coverings of curves.

(English) Zbl 1365.14048
J. Korean Math. Soc. 54, No. 1, 69-86 (2017).

Let C be a non-singular irreducible algebraic curve of genus $g \geq 2$ defined over a field K of characteristic 0. The notion of *Galois Weierstrass points* was introduced by *I. Morrison* and *H. Pinkham* in [Ann. Math. (2) 124, 591–625 (1986; Zbl 0624.14018)]. In particular, a point $P \in C$ is said to be a Galois Weierstrass point (GW point) if $\Phi_{|aP|} : C \to \mathbb{P}^1$ is a Galois covering, where a is the smallest positive integer of the Weierstrass semigroup

$$H(P) = \{n \in \mathbb{Z}_{\geq 0} \mid \exists f \in K(C) \text{ with } (f)_\infty = nP\}.$$

In general, a GW point is not necessary a Weierstrass point. In this paper points which are both GW and Weierstrass are considered. The definition of a *Galois point* for plane curves of degree $d \geq 3$ is due to *H. Yoshihara* [J. Algebra 239, No. 1, 340–355 (2001; Zbl 1064.14023)]. A point $P \in \mathbb{P}^2$ is a Galois point for C if the extension of function fields $K(\mathbb{P}^1) \subseteq K(C)$ induced by the projection ${\pi_P}^* : C \to \mathbb{P}^1$ is Galois. In particular P is said to be an inner (resp. outer) Galois point if $P \in C$ (resp. $P \notin C$).

In this paper, the relationship between certain kind of GW points and Galois points is studied by examining the branch points of double coverings of curves. Some studies on double covering of curves have be conducted in [*F. Torres*, Manuscr. Math. 83, No. 1, 39–58 (1994; Zbl 0838.14025)] and [*S. J. Kim* and *J. Komeda*, J. Algebra 322, No. 1, 137–152 (2009; Zbl 1171.14020)]. To do that two new definitions are given as follows:

A point $P \in C$ is a *Weak GW point* if P is totally ramified in some Galois covering $f : C \to \mathbb{P}^1$ and P is a Weieratrass point of C. If P is not a GW point, then it is referred as a *Pseudo-GW point*. The *set of degrees of the Weak GW point* P is $\mathrm{degGW}(P) = \{\deg f \mid P \text{ is totally ramified in the Galois covering } f : C \to \mathbb{P}^1\}$. Moreover, the main result of this paper is stated in the following theorem.

Theorem. Let C be a plane curve of degree $d \geq 5$ such that there exists a Galois point P for C and a double covering $\phi C \to C'$, where C' is a curve and $\mathrm{Ramif}(\phi_P) \cap \mathrm{Ramif}(\phi) \neq \emptyset$. Choose $P' \in C'$ as follows:

(1) if P is an inner Galois point, then $P' := \phi(P)$.

(2) if P is an outer Galois point, then let $P' = \phi(Q)$ for some $Q \in \mathrm{Ramif}(\phi_P) \cap \mathrm{Ramif}(\phi)$.

Then, P' is a weak GW point. Moreover, the main results on $H(P')$ and the set of degrees of the Weak GW point of P' with respect to d and P are summarized in the following table.

	P is an inner Galois point	P is an outer Galois point
d is odd	$H(P') = \langle \frac{d-1}{2}, d \rangle$, $\frac{d-1}{2} \in \mathrm{degGW}(P')$	$H(P') = \langle \frac{d-q}{2}, d \rangle$, $d \in \mathrm{degGW}(P')$
d is even	$H(P') = \langle \frac{d}{2}, d-1 \rangle$, $d-1 \in \mathrm{degGW}(P')$	$H(P') = \langle \frac{d}{2}, d-1 \rangle$, $\frac{d}{2} \in \mathrm{degGW}(P')$

Conversely, let C' be a curve and P' a weak GW point with $H(P') = \langle \frac{d-1}{2}, d \rangle$ or $\langle \frac{d}{2}, d-1 \rangle$ for $d \geq 5$. Then, there exists a plane curve C of degree d, a Galois point P for C and a double covering $\phi : C \to C'$ such that P' is obtained as above.

Reviewer: Maria Montanucci (Umbertide)

MSC:

14H55 Riemann surfaces; Weierstrass points; gap sequences
14H05 Algebraic functions and function fields in algebraic geometry
14H37 Automorphisms of curves
14H50 Plane and space curves

Keywords:

Galois point; Galois Weierstrass point; weak Galois Weierstrass point; pseudo-Galois Weierstrass point

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