Dykema, Ken; Sukochev, Fedor; Zanin, Dmitriy

Algebras of log-integrable functions and operators. (English) Zbl 1373.46059

Complex Anal. Oper. Theory 10, No. 8, 1775-1787 (2016).

Let (Ω,ν) be a measure space, the function space $\mathcal{L}_{\log}(\Omega,\nu)$ consists of measurable functions f such that $\int_\Omega \log(1+|f|)\,d\nu<\infty$. A non-commutative operator algebra version $\mathcal{L}_{\log}(\mathcal{M},\tau)$ over a von Neumann algebra $\mathcal{M}$ with a normal, faithful, semifinite trace τ is defined to be the set of all τ-measurable operators affiliated with $\mathcal{M}$ such that $\tau(\log(1+|T|))<\infty$. In this paper, the authors show that they are complete topological *-algebras, with respect to the F-norms $\|f\|_{\log}:=\int_\Omega \log(1+|f|)\,d\nu$ and $\|T\|_{\log}:=\tau(\log(1+|T|))$, respectively. Note that the authors treat the commutative case $\mathcal{L}_{\log}(\Omega,\nu)$ separately, although it is a special case of $\mathcal{L}_{\log}(\mathcal{M},\tau)$, they show that the space $\mathcal{L}_{\log}(\Omega,\nu)$ is a non-locally convex generalised Orlicz space and that the F-norm $\|\cdot\|_{\log}$ is equivalent to the one given in [*W. Matuszewska* and *W. Orlicz*, Stud. Math. 21, 107–115 (1961; Zbl 0202.39903)]. The authors also explore connections with the Nevanlinna class of holomorphic functions on the open unit disk.

Reviewer: Ying-Fen Lin (Belfast)

MSC:

46L52 Noncommutative function spaces
46H35 Topological algebras of operators
46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
30H15 Nevanlinna spaces and Smirnov spaces

Keywords:
log-integrable functions; measurable operators; complete *-algebras; Nevanlinna class

Full Text: DOI arXiv

References:

[1] Brown, L.: Lidskii's theorem in the type II case, geometric methods in operator algebras, Kyoto. Pitman Res. Notes Math. Ser., 123, Longman Sci. Tech. Harlow 1986, 1-35 (1983)

[2] Dodds, PG; Dodds, TK-Y; de Pagter, B, Noncommutative Köthe duality, Trans. Amer. Math. Soc., 339, 717-750, (1993) · Zbl 0801.46074

[3] Duren, P.L.: Theory of H^{p} spaces. Pure Appl. Math. \textbf{38}, Academic Press, New York-London, (1970) · Zbl 0215.20203

[4] Dykema, K., Sukochev, F., Zanin, D.: An upper triangular decomposition theorem for some unbounded operators affiliated to II$_1$-factors. arXiv:1509.03358 · Zbl 06821077

[5] Fack, T; Kosaki, H, Generalized s-numbers of $τ $-measurable operators, Pacific J. Math., 123, 269-300, (1986) · Zbl 0617.46063

[6] Fuglede, B; Kadison, R, Determinant theory in finite factors, Ann. of Math., 55, 520-530, (1952) · Zbl 0046.33604

[7] Haagerup, U; Schultz, H, Brown measures of unbounded operators affiliated with a finite von Neumann algebra, Math. Scand., 100, 209-263, (2007) · Zbl 1168.46039

[8] Kalton, N.J., Peck, N.T., Roberts, J.W.: An F-space sampler, London Mathematical Society Lecture Note Series, 89, Cambridge University Press, Cambridge (1984) · Zbl 0202.39903

[9] Maligranda, L.: Orlicz spaces and interpolation, Seminários de Matemática, vol., 5, Universidade Estadual de Campinas, Departamento de Matemática, Campinas (1989) · Zbl 0874.46022

[10] Matuszewska, W; Orlicz, W, A note on the theory of s-normed spaces of $φ $-integrable functions, Studia Math., 21, 107-115, (1962) · Zbl 0202.39903

[11] Nelson, E, Notes on non-commutative integration, J. Funct. Anal., 15, 103-116, (1974) · Zbl 0292.46030

[12] Rudin, W.: Functional analysis, McGraw-Hill series in higher mathematics. McGraw-Hill Book Co., New York-Düsseldorf-Johannesburg (1973)

[13] Shapiro, J.H., Shields, A.L.: Unusual topological properties of the Nevanlinna class. Amer. J. Math. \textbf{97}, 915-936 (1975) · Zbl 0323.30033

[14] Sinclair, A.M., Smith, R.R.: Finite von Neumann algebras and masas, London mathematical society lecture note series, 351. Cambridge University Press, Cambridge (2008)

[15] Sukochev, F.A.: Order properties of norms of symmetric spaces of measurable operators, mathematical analysis and probability theory, Tashkent. Gos. Univ., Tashkent. \textbf{94}, 49-54 (1985)