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A first class constraint generates not a gauge transformation, but a bad physical change: The case of electromagnetism. (English) Zbl 1360.70030

Ann. Phys. 351, 382-406 (2014).

Summary: In Dirac-Bergmann constrained dynamics, a first-class constraint typically does not *alone* generate a gauge transformation. By direct calculation it is found that each first-class constraint in Maxwell's theory generates a change in the electric field $\vec{E}$ by an arbitrary gradient, spoiling Gauss's law. The secondary first-class constraint $p^i{}_{,i} = 0$ still holds, but being a function of derivatives of momenta (mere auxiliary fields), it is not directly about the observable electric field (a function of derivatives of A_μ), which couples to charge. Only a special combination of the two first-class constraints, the Anderson-Bergmann-Castellani gauge generator G, leaves $\vec{E}$ unchanged. Likewise only that combination leaves the canonical action invariant – an argument independent of observables. If one uses a first-class constraint to generate instead a canonical transformation, one partly strips the canonical coordinates of physical meaning as electromagnetic potentials, vindicating the Anderson-Bergmann Lagrangian orientation of interesting canonical transformations. The need to keep gauge-invariant the relation $\dot{q} - \frac{\delta H}{\delta p} = -E_i - p^i = 0$ supports using the gauge generator and primary Hamiltonian rather than the separate first-class constraints and the extended Hamiltonian.Partly paralleling Pons's criticism, it is shown that Dirac's proof that a first-class primary constraint generates a gauge transformation, by comparing evolutions from *identical* initial data, cancels out and hence fails to detect the alterations made to the initial state. It also neglects the arbitrary coordinates multiplying the secondary constraints *inside* the canonical Hamiltonian. Thus the gauge-generating property has been ascribed to the primaries alone, not the primary-secondary team G. Hence the Dirac conjecture about secondary first-class constraints as generating gauge transformations rests upon a false presupposition about primary first-class constraints. Clarity about Hamiltonian electromagnetism will be useful for an analogous treatment of GR.

MSC:

70H45	Constrained dynamics, Dirac's theory of constraints
70S15	Yang-Mills and other gauge theories in mechanics of particles and systems
70S05	Lagrangian formalism and Hamiltonian formalism in mechanics of particles and systems
78A25	Electromagnetic theory, general
81S05	Commutation relations and statistics as related to quantum mechanics (general)

Keywords:

Dirac-Bergmann constrained dynamics; Gauge transformations; canonical quantization; observables; Hamiltonian methods; first-class constraints

Full Text: DOI arXiv

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