

**Murakami, Jun**

**Generalized Kashaev invariants for knots in three manifolds.** (English) Zbl 1365.57020  
*Quantum Topol.* 8, No. 1, 35-73 (2017).

The aim of the paper is a generalization of Kashaev's invariants for a knot in the 3-sphere to invariants of a knot in a 3-manifold. From a study of quantum dilogarithms, Kashaev introduced an invariant of links in 3-manifolds, gave an  $R$ -matrix formulation of his invariant for knots in the 3-sphere and found a relation between his invariant and the hyperbolic volume of knot complements in  $S^3$ , resulting in a conjecture not yet rigorously proved. A complexified refinement of Kashaev's conjecture for hyperbolic knots in the 3-sphere, in terms of the hyperbolic volume and the Chern-Simons invariant, has been given by *H. Murakami* et al. [*Exp. Math.* 11, No. 3, 427-435 (2002; [Zbl 1117.57300](#))].

"The aim of the present paper is to construct certain quantum invariants for knots in 3-manifolds which have a relation to the hyperbolic volume as the above conjectures." "We construct a family of invariants of a knot in a 3-manifold by combining the invariant of 3-manifolds by *M. Hennings* [*J. Lond. Math. Soc.*, II. Ser. 54, No. 3, 594-624 (1996; [Zbl 0882.57002](#))] and the logarithmic invariant of knots in  $S^3$  [*J. Murakami* and *K. Nagamoto*, *Int. J. Math.* 19, No. 10, 1203-1213 (2008; [Zbl 1210.57016](#))]. This family contains a generalized Kashaev invariant which coincides with Kashaev's invariant for the case of the 3-sphere."

The author states a volume conjecture for his generalized Kashaev invariant, again in terms of the volume of a hyperbolic knot in a 3-manifold and the Chern-Simons invariant, and checks the conjecture numerically for some knots in lens spaces.

Reviewer: [Bruno Zimmermann \(Trieste\)](#)

**MSC:**

- 57M27 Invariants of knots and 3-manifolds (MSC2010)
- 16T05 Hopf algebras and their applications
- 17B37 Quantum groups (quantized enveloping algebras) and related deformations
- 81R50 Quantum groups and related algebraic methods applied to problems in quantum theory

Cited in **2** Reviews  
Cited in **7** Documents

**Keywords:**

[knot in a 3-manifold](#); [hyperbolic knot](#); [volume](#); [Kashaev invariant](#); [volume conjecture](#)

**Full Text:** [DOI](#) [arXiv](#)

**References:**

- [1] pp.

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