**Fawzi, Hamza**; **Saunderson, James**; **Parrilo, Pablo A.**
**Equivariant semidefinite lifts of regular polygons.** (English) Zbl 1364.90245
Math. Oper. Res. 42, No. 2, 472-494 (2017).

Summary: Given a polytope $P \subset \mathbb{R}^n$, we say that $P$ has a positive semidefinite lift (psd lift) of size $d$ if one can express $P$ as the projection of an affine slice of the $d \times d$ positive semidefinite cone. Such a representation allows us to solve linear optimization problems over $P$ using a semidefinite program of size $d$ and can be useful in practice when $d$ is much smaller than the number of facets of $P$. If a polytope $P$ has symmetry, we can consider equivariant psd lifts, i.e., those psd lifts that respect the symmetries of $P$. One of the simplest families of polytopes with interesting symmetries is regular polygons in the plane. In this paper, we give tight lower and upper bounds on the size of equivariant psd lifts for regular polygons. We give an explicit construction of an equivariant psd lift of the regular $2^n$-gon of size $2n-1$, and we prove that our construction is essentially optimal by proving a lower bound on the size of any equivariant psd lift of the regular $N$-gon that is logarithmic in $N$. Our construction is exponentially smaller than the (equivariant) psd lift obtained from the Lasserre/sum-of-squares hierarchy, and it also gives the first example of a polytope with an exponential gap between equivariant psd lifts and equivariant linear programming lifts.

**MSC:**

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|---|---|
| 90C22 | Semidefinite programming |
| 52B15 | Symmetry properties of polytopes |
| 68Q17 | Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.) |



**Keywords:**

semidefinite programming; extended formulations; sums of squares

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