Ball, Keith

Volumes of sections of cubes and related problems. (English) Zbl 0674.46008

Geometric aspects of functional analysis, Isr. Semin., GAFA, Isr. 1987-88, Lect. Notes Math. 1376, 251-260 (1989).

[For the entire collection see Zbl 0668.00010.]

The volume ratio of a symmetric convex body $C \subset \mathbb{R}^n$ is the number

$$vr(C) = (vol \quad C/vol \quad E)^{1/n},$$

where vol C = volume of C and vol E = volume of the ellipsoid E of maximum volume contained in C. A classical result of F. John shows that $vr(c) < \sqrt{n}$ for every such symmetric n-dimensional convex body C.

In this paper, it is shown that $vr(C) \le vr(Q_n) = (2/V_e^{1/2})$, where Q_n is the n-dimensional cube in $\mathbb{R}^n$ and V_n is the volume of the unit ball in $\mathbb{R}^n$. A reformulation of this result in terms of volumes of sections of Q_n, a derivation of the best possible upper bound for the volume of such a section, and an "isomorphic version" of the above inequality which involves the so- called cubical volume ratio of C are also given.

Reviewer: J.R.Holub

MSC:

46B20	Geometry and structure of normed linear spaces	
52C07	Lattices and convex bodies in n dimensions (aspects of discrete geometry)	

Keywords:
volume ratio of a symmetric convex body; isomorphic version; cubical volume ratio