Cianchi, Andrea

On relative isoperimetric inequalities in the plane. (English) Zbl 0674.49030

Boll. Unione Mat. Ital., VII. Ser., B 3, No. 2, 289-325 (1989).

An isoperimetric inequality relative to an open bounded subset G of $\mathbb{R}^n$ is said to hold if two positive constants and Q exist such that

$$(1) \quad P(E;G) \quad Q \geq [\min\{measE, \quad measG \setminus E\}]^{\alpha}$$

for every measurable subset E of G. Here "meas" is Lebesgue measure and P(E;G) is the perimeter of E relative to G. The smallest number Q for which (1) holds is called the isoperimetric constant relative to G and will be denoted by Q(α ;G). In our paper we consider the case $n = 2$. We show that, if $G \subset \mathbb{R}^2$ is convex, then there exist subsets of G satisfying the equality in (1) with $Q = Q(\alpha; G)$ and we give a characterization of such subsets. The isoperimetric constants relative to triangles, regular polygons and convex sets with a center of symmetry are computed. Finally, we get sharp estimates for Q(α ;G) as G belongs to particular classes of convex sets.

Reviewer: A.Cianchi

MSC:

49Q05 Minimal surfaces and optimization
49Q15 Geometric measure and integration theory, integral and normal currents in optimization

Keywords:

isoperimetric inequality; triangles; regular polygons; convex sets