Summary: We investigate the geometric theory of local MV-algebras and its quotients axiomatizing the local MV-algebras in a given proper variety of MV-algebras. We show that, whilst the theory of local MV-algebras is not of presheaf type, each of these quotients is a theory of presheaf type which is Morita-equivalent to an expansion of the theory of lattice-ordered abelian groups. A. Di Nola and A. Lettieri's [Stud. Log. 53, No. 3, 417–432 (1994; Zbl 0812.06010)] equivalence is recovered from the Morita-equivalence for the quotient axiomatizing the local MV-algebras in C. C. Chang's variety [Trans. Am. Math. Soc. 88, 467–490 (1958; Zbl 0084.00704)], that is, the perfect MV-algebras. We establish along the way a number of results of independent interest, including a constructive treatment of the radical for MV-algebras in a fixed proper variety of MV-algebras and a representation theorem for the finitely presentable algebras in such a variety as finite products of local MV-algebras.

MSC:

06D35 MV-algebras
03G30 Categorical logic, topoi
06B20 Varieties of lattices
06F20 Ordered abelian groups, Riesz groups, ordered linear spaces
18B25 Topoi
18C10 Theories (e.g., algebraic theories), structure, and semantics

Keywords:
local MV-algebra; lattice-ordered abelian group; Morita-equivalence; geometric logic; classifying topos; Grothendieck topology; theory of presheaf type

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References:
