Let $\{F_{n}^{(k)}(x)\}_{n=0}^{\infty}$ be the sequence of Fibonacci-type polynomials of order $k$ of the authors and G. N. Philippou [ibid. 23, 100-105 (1985; Zbl 0563.10014)], and let $\{F_{n,r}^{(k)}(x)\}_{n=0}^{\infty}$ be the (r-1)-fold convolution of $\{F_{n}^{(k)}(x)\}_{n=0}^{\infty}$ with itself. The authors derive multinomial and binomial expansions for $F_{n+1,r}^{(k)}(x)$, $n \geq 0$, as well as the following recurrence:

**Theorem 1.** Set $F_{0,r}^{(k)}(x) = 0$ for $-k + 1 \leq n \leq -1$. Then $F_{0,r}^{(k)}(x) = 0$, $F_{1,r}^{(k)}(x) = 1$, and

$$F_{n+1,r}^{(k)}(x) = n^{-1}x \sum_{j=1}^{k} [n + j(r - 1)]F_{n+1-j,r}^{(k)}(x), \quad n \geq 1.$$ 

Using the above mentioned results, the authors derive binomial expressions for the negative binomial distribution of order $k$ of the authors and G. N. Philippou [Stat. Probab. Lett. 1, 171-175 (1983; Zbl 0517.60010)], say $NB_{k,1}(r, p)$, and the negative binomial (or compound Poisson) distribution of order $k$ of the first author [Zap. Nauchn. Semin. Leningr. Otd. Mat. Inst. Steklova 130, 175-180 (1983; Zbl 0529.60010)], say $NB_{k,II}(r, p) = CP_{k}(r, \alpha)$ with $\alpha = kp/q$. They also derive the following useful recurrences for calculating probabilities in these two distributions.

**Theorem 2.** Let $X$ be a random variable distributed as $NB_{k,1}(r, p)$, and set $P_{n} = P(X = n)$. Then

$$P_{n} = 0 \quad for \quad n \leq kr - 1, \quad P_{n} = p^{kr} \quad for \quad n = kr, \quad and$$

$$P_{n} = q^{-1}(n - kr)^{-1} \sum_{j=1}^{k} [n - kr + j(r - 1)]p^{j}P_{n-j} \quad for \quad n \geq kr + 1.$$ 

**Theorem 3.** Let $X$ be a random variable distributed as $NB_{k,II}(r, p)$, and set $P_{n} = P(X = n)$. Then

$$P_{n} = 0 \quad for \quad -k + 1 \leq n \leq -1, \quad P_{n} = p^{r} \quad for \quad n = 0, \quad and$$

$$P_{n} = qk^{-1}n^{-1} \sum_{j=1}^{k} [n + j(r - 1)]P_{n-j} \quad for \quad n \geq 1.$$ 

The computational usefulness of the binomial expressions is illustrated by two examples. (Reviewer’s remark: In Definition 3.2, q/p should be replaced by q/k).

Reviewer: A.N. Philippou