Philippou, Andreas N.; Georghiou, Costas
Convolutions of Fibonacci-type polynomials of order \( k \) and the negative binomial distributions of the same order. (English) Zbl 0677.60012

Let \( \{F_n^{(k)}(x)\}_{n=0}^{\infty} \) be the sequence of Fibonacci-type polynomials of order \( k \) of the authors and G. N. Philippou [ibid. 23, 100-105 (1985; Zbl 0563.10014)], and let \( \{F_n^{(k)}(x)\}_{n=0}^{\infty} \) be the (r-1)-fold convolution of \( \{F_n^{(k)}(x)\}_{n=0}^{\infty} \) with itself. The authors derive multinomial and binomial expansions for \( F_{n+1,r}^{(k)}(x), n \geq 0 \), as well as the following recurrence:

**Theorem 1.** Set \( F_{0,r}^{(k)}(x) = 0 \) for \( -k+1 \leq n \leq -1 \). Then \( F_{0,r}^{(k)}(x) = 0, F_{1,r}^{(k)}(x) = 1 \), and

\[
F_{n+1,r}^{(k)}(x) = n^{-1}x \sum_{j=1}^{k} [n+j(r-1)]F_{n+1-j,r}^{(k)}(x), \quad n \geq 1.
\]

Using the above mentioned results, the authors derive binomial expressions for the negative binomial distribution of order \( k \) of the authors and G. N. Philippou [Stat. Probab. Lett. 1, 171-175 (1983; Zbl 0517.60010)], say \( \text{NB}_{k,1}(r,p) \), and the negative binomial (or compound Poisson) distribution of order \( k \) of the first author [Zap. Nauchn. Semin. Leningr. Otd. Mat. Inst. Steklova 130, 175-180 (1983; Zbl 0529.60010)], say \( \text{NB}_{k,II}(r,p) = \text{CP}_k(r,\alpha) \) with \( \alpha = kp/q \). They also derive the following useful recurrences for calculating probabilities in these two distributions.

**Theorem 2.** Let \( X \) be a random variable distributed as \( \text{NB}_{k,1}(r,p) \), and set \( P_n = P(X = n) \). Then

\[
P_n = 0 \quad \text{for} \quad n \leq kr - 1, \quad P_n = p^{kr} \quad \text{for} \quad n = kr, \quad \text{and}
\]

\[
P_n = qp^{-1}(n-kr)^{-1} \sum_{j=1}^{k} [n-kr+j(r-1)]p^j P_{n-j} \quad \text{for} \quad n \geq kr + 1.
\]

**Theorem 3.** Let \( X \) be a random variable distributed as \( \text{NB}_{k,II}(r,p) \), and set \( P_n = P(X = n) \). Then

\[
P_n = 0 \quad \text{for} \quad -k+1 \leq n \leq -1, \quad P_n = p^{r} \quad \text{for} \quad n = 0, \quad \text{and}
\]

\[
P_n = qk^{-1}n^{-1} \sum_{j=1}^{k} [n+j(r-1)]P_{n-j} \quad \text{for} \quad n \geq 1.
\]

The computational usefulness of the binomial expressions is illustrated by two examples. (Reviewer’s remark: In Definition 3.2, \( q/p \) should be replaced by \( q/k \)).

Reviewer: A.N.Philippou

**MSC:**

60E05 Probability distributions: general theory
11B37 Recurrences
11B39 Fibonacci and Lucas numbers and polynomials and generalizations

**Keywords:**

Fibonacci-type polynomials; negative binomial distribution