

Barthel, Tobias; Heard, Drew; Valenzuela, Gabriel

Local duality for structured ring spectra. (English) Zbl 1384.55008
J. Pure Appl. Algebra 222, No. 2, 433-463 (2018).

In earlier work [“Local duality in algebra and topology”, Preprint, [arXiv:1511.03526](https://arxiv.org/abs/1511.03526)] the authors of the present paper studied local homology and local cohomology functors in the context of stable ∞ -categories. In the paper under review, they apply a slightly generalized version of their theory to the study of structured ring spectra. For example, given an E_∞ ring spectrum R with $\pi_*(R)$ Noetherian and a specialization closed subset of the Zariski spectrum of $\pi_*(R)$, they produce a “local duality context” consisting of four endofunctors of Mod_R satisfying various properties.

The results of the present paper recover and generalize various results about local cohomology studied in the literature. For example, the authors show that their theory recovers local homology and local cohomology functors studied by *D. Benson* et al. [Ann. Sci. Éc. Norm. Supér. (4) 41, No. 4, 575–621 (2008; [Zbl 1171.18007](#))] and provides a better understanding of certain spectral sequences related to these functors. As another application, they study a Gorenstein condition for structured ring spectra that is related to analogous notions appearing in work of *W. G. Dwyer* et al. [Adv. Math. 200, No. 2, 357–402 (2006; [Zbl 1155.55302](#))]. Based on this, they study a notion of twisted Gorenstein duality that leads to a generalization of work of *D. J. Benson* and *J. P. C. Greenlees* [J. Pure Appl. Algebra 212, No. 7, 1716–1743 (2008; [Zbl 1161.20005](#))] when applied to the cochain algebra of the classifying space of a compact Lie group with coefficients in a field.

Reviewer: [Steffen Sagave \(Nijmegen\)](#)

MSC:

- [55P43](#) Spectra with additional structure (E_∞ , A_∞ , ring spectra, etc.)
- [14B15](#) Local cohomology and algebraic geometry
- [13D45](#) Local cohomology and commutative rings

Cited in **5** Documents

Keywords:

Structured ring spectrum; local duality; local cohomology; Gorenstein duality

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