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Moment conditions in strong laws of large numbers for multiple sums and random measures.

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Summary: The validity of the strong law of large numbers for multiple sums $S_{\mathbf{n}}$ of independent identically distributed random variables $Z_{\mathbf{k}}$, $\mathbf{k} \leq \mathbf{n}$, with r -dimensional indices is equivalent to the integrability of $|Z|(\log^+ |Z|)^{r-1}$, where Z is the generic summand. We consider the strong law of large numbers for more general normalizations, without assuming that the summands $Z_{\mathbf{k}}$ are identically distributed, and prove a multiple sum generalization of the Brunk-Prohorov strong law of large numbers. In the case of identical finite moments of order $2q$ with integer $q \geq 1$, we show that the strong law of large numbers holds with the normalization $(n_1 \cdots n_r)^{1/2}(\log n_1 \cdots \log n_r)^{1/(2q)+\varepsilon}$ for any $\varepsilon > 0$.

The obtained results are also formulated in the setting of ergodic theorems for random measures, in particular those generated by marked point processes.

MSC:

[60G55](#) Point processes (e.g., Poisson, Cox, Hawkes processes)

[60F15](#) Strong limit theorems

[60D05](#) Geometric probability and stochastic geometry

Keywords:

[multiple sums of random variables](#); [multiindices](#); [strong law of large numbers](#); [random measures](#)

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