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The frontier between geometry and physics. (English) Zbl 0679.53074

Jahresber. Dtsch. Math.-Ver. 91, No. 4, 149-158 (1989).

This article on the interaction between geometry and physics is partly expository, and partly philosophical. The overall tone is set by the following sentences in the introduction: "Geometrical ideas are used as a basis of physics models. New ideas and techniques of a physical character then naturally emerge and some of these may feed back into geometry, suggesting new points of view". In order to illustrate this state of affairs two specific topics are selected. The first of these is the theory of polynomial invariants for knots and links in 3-space, as initiated by *V. F. R. Jones* [Bull. Am. Math. Soc., New Ser. 12, 103-111 (1985; Zbl 0564.57006)]. It is indicated how this theory is related to certain important models in statistical mechanics; this has had far-reaching consequences. Moreover, further series of polynomial invariants for links have since been constructed, and, according to the author, there are indications that there should be one such invariant for each pair (G,V), where G is a compact Lie group and V an irreducible representation. (The original polynomial of Jones corresponds to the case $G = U(2)$, with V identified with the basic 2-dimensional representation.)

The second topic is concerned with the recent results on the structure 4- manifolds due to S. K. Donaldson. After a brief summary of some of the corresponding material various methods which had been used in this analysis are described, with special emphasis on those methods that have their origins in physics. This involves the instaton solutions of self- dual Yang-Mills equations, and it is indicated how this can be related to specific quantum effects. In the concluding section the following observations are presented: "The really significant fact about the relation between geometry and physics in all this is that it is quantum physics which is related to topological aspects of geometry. In contrast the Einstein theory is a relation at the classical level between gravitation and local differential geometry. It is only at the quantum level that global topological phenomena become relevant".

Reviewer: H.Rund

MSC:

53C80 Applications of global differential geometry to the sciences
22E70 Applications of Lie groups to the sciences; explicit representations
57M25 Knots and links in the 3-sphere (MSC2010)
58J10 Differential complexes
81T08 Constructive quantum field theory

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polynomial invariants for knots and links; differentiable 4-manifolds; Yang-Mills instantons