

Ablinger, J.; Behring, A.; Blümlein, J.; De Freitas, A.; von Manteuffel, A.; Schneider, C.
The three-loop splitting functions $P_{qq}^{(2)}$ and $P_{gg}^{(2, N_F)}$. (English) [Zbl 1373.81370](#)
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Summary: We calculate the unpolarized twist-2 three-loop splitting functions $P_{qq}^{(2)}(x)$ and $P_{gg}^{(2, N_F)}(x)$ and the associated anomalous dimensions using massive three-loop operator matrix elements. While we calculate $P_{gg}^{(2, N_F)}(x)$ directly, $P_{qq}^{(2)}(x)$ is computed from 1200 even moments, without any structural prejudice, using a hierarchy of recurrences obtained for the corresponding operator matrix element. The largest recurrence to be solved is of order 12 and degree 191. We confirm results in the foregoing literature.

MSC:

- 81V05** Strong interaction, including quantum chromodynamics
- 81T15** Perturbative methods of renormalization applied to problems in quantum field theory
- 65Q30** Numerical aspects of recurrence relations

Cited in **6** Documents

Software:

[Axodraw](#); [EvaluateMultiSums](#); [FORM](#) ; [HarmonicSums](#); [Reduze](#)

Full Text: [DOI](#)

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