Pellegrino, D.; Santos, J.; Serrano-Rodríguez, D.; Teixeira, E.

A regularity principle in sequence spaces and applications. (English) Zbl 1404.46041

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The authors prove a rather general regularity principle, from which they deduce several results in different directions: inclusion theorems for multiple summing operators, Grothendieck-type theorems and Hardy-Littlewood inequalities. We emphasise two of them.

On the one hand it is shown that every m-linear mapping $T:\ell_1\times\cdots\times\ell_1\to\ell_2$ is multiple $(q;p)$-summing if and only if $p\leq 2$ or $q>p>2$.

The second result that we emphasise is a version of the Hardy-Littlewood inequality for bilinear forms on $\ell_p\times\ell_q$. Consider $2\leq p,q\leq\infty$ with $\frac{1}{p}+\frac{1}{q}<1$ and $s,t>0$. There exists a constant $C>0$ such that

$$\left(\sum_{i=1}^{n}\left(\sum_{j=1}^{n}|T(e_i,e_j)|^s\right)^{\frac{t}{s}}\right)^{\frac{1}{t}}\leq C\|T\|$$

for every $T:\ell_p^n\times\ell_q^n\to\mathbb{K}$ (note that C depends on p,q,s,t but not on n) if and only if the following three conditions are satisfied: $\frac{q}{q-1}\leq s<\infty$, $\frac{pq}{pq-p-q}\leq t<\infty$ and $\frac{1}{s}+\frac{1}{t}\leq\frac{3}{2}-\left(\frac{1}{p}+\frac{1}{q}\right)$.

If these conditions are not satisfied, then the constant C in the inequality depends on n. A careful study is performed, giving the precise asymptotic growth on n of the constant for fixed p,q,s,t.

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MSC:

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46B45 Banach sequence spaces
47L22 Ideals of polynomials and of multilinear mappings in operator theory

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