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**Space-time asymptotics of the two dimensional Navier-Stokes flow in the whole plane.** (English) [Zbl 1378.35225](#)

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Summary: We consider the space-time behavior of the two dimensional Navier-Stokes flow. Introducing some qualitative structure of initial data, we succeed to derive the first order asymptotic expansion of the Navier-Stokes flow without moment condition on initial data in  $L^1(\mathbb{R}^2) \cap L^2_\sigma(\mathbb{R}^2)$ . Moreover, we characterize the necessary and sufficient condition for the rapid energy decay  $\|u(t)\|_2 = o(t^{-1})$  as  $t \rightarrow \infty$  motivated by *T. Miyakawa* and *M. E. Schonbek* [Math. Bohem. 126, No. 2, 443-455 (2001; [Zbl 0981.35048](#))]. By weighted estimated in Hardy spaces, we discuss the possibility of the second order asymptotic expansion of the Navier-Stokes flow assuming the first order moment condition on initial data. Moreover, observing that the Navier-Stokes flow  $u(t)$  lies in the Hardy space  $H^1(\mathbb{R}^2)$  for  $t > 0$ , we consider the asymptotic expansions in terms of Hardy-norm. Finally we consider the rapid time decay  $\|u(t)\|_2 = o(t^{-\frac{3}{2}})$  as  $t \rightarrow \infty$  with cyclic symmetry introduced by *L. Brandolese* [C. R. Acad. Sci., Paris, Sér. I, Math. 332, No. 2, 125-130 (2001; [Zbl 0973.35149](#))].

**MSC:**

[35Q30](#) Navier-Stokes equations  
[76D05](#) Navier-Stokes equations for incompressible viscous fluids  
[35C20](#) Asymptotic expansions of solutions to PDEs  
[35B40](#) Asymptotic behavior of solutions to PDEs  
[30H10](#) Hardy spaces

Cited in 1 Document

**Keywords:**

time decay; asymptotic expansion; large time behavior

**Full Text:** [DOI](#)

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