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Asymptotics of polynomials of simultaneous orthogonality in Angelesko's case. (Russian)

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Mat. Sb., N. Ser. 136(178), No. 1(5), 56-84 (1988).

Let ρ_i be a weight (integrable a.e. positive) function on the finite segment Δ_i, $i = 1, 2, ..., p$, where $\Delta_i \cap \Delta_j = \emptyset$, $i \neq j$. For a fixed set of natural numbers $\bar{n} = (n_1, n_2, ..., n_p)$, $Q_n(z) = z^{|\bar{n}|} + ...$ is called a polynomial of simultaneous orthogonality with respect to $(\rho_1, ..., \rho_p)$ if $\deg Q_{\bar{n}} \leq |\bar{n}| = n_1 + n_2 + ... + n_p$ and $\int_{\Delta_i} Q_{\bar{n}}(x) x^{\nu} \rho_i(x) dx = 0$, $\nu = 0, 1, ..., n_i - 1$ $(i = 1, ..., p)$. If $p = 1$ we obtain classical orthogonality relations. In this case, if $\rho = \rho_1$ satisfies Szegö's condition, where $\Delta_1 = [-1, 1]$, it is well known that the sequence of polynomials $\{Q_n\}$, $n \in \mathbb{N}$, satisfy Bernstein-Szegö's asymptotic formula

$$Q_n(z) = ()^n \phi_n(z)(F(z) + o(1)), \quad n \to \infty, \quad z \in \bar{\mathbb{C}} \setminus [-1, 1],$$

where $\phi(z) = z + \sqrt{z^2 - 1}$ and F is an analytic function on $\bar{\mathbb{C}} \setminus [-1, 1]$, $F(\infty) = 1$, whose boundary values are determined by ρ (x). The author ingeneously combines techniques of the theories of Riemann surfaces and extremal boundary problems of analytic functions with theoretical- potential arguments to obtain an extension of Bernstein-Szegö's classical formula in the general setting described above of simultaneous orthogonal polynomials. These results find applications in Hermite-Padé approximation, diophantine approximation and in the construction of the spectral and scattering theories of difference operators of order $p + 1$.

Reviewer: G.López Lagomasino

MSC:

42C05	Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
33C45	Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
41A60	Asymptotic approximations, asymptotic expansions (steepest descent, etc.)

Keywords:

weight; simultaneous orthogonality; Bernstein-Szegö's asymptotic formula; Riemann surfaces; Hermite-Padé approximation; diophantine approximation

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