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Representation of integers as sums of fractional powers of primes and powers of 2. (English)

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The binary Goldbach conjecture, which states that every even integer $N \geq 4$ should be representable as a sum of two primes, is still wide-open. However, *Yu. V. Linnik* [Dokl. Akad. Nauk SSSR, n. Ser. 85, 953–954 (1952; Zbl 0047.04505)] established that every large enough even integer can be represented as the sum of two primes and a bounded number of powers of 2. *J. Liu* et al. [J. Number Theory 92, No. 1, 99–116 (2002; Zbl 0997.11082)] proved a variant of this result, namely that every large enough even integer N can be represented as the sum of four squares of primes and a bounded number of powers of 2. The article under review considers another interesting variant of Linnik's result, namely representations of the form

$$N = [p_1^c] + [p_2^c] + 2^{\nu_1} + \cdots + 2^{\nu_k},$$

where p_1 and p_2 are primes and c is a real number greater 1. It is proved that for any $c \in (1, 29/28)$, there exists an integer k such that every large enough integer N can be represented in the form above. The author's treatment is based on the circle method and uses, among other tools, exponential sum and integral estimates, zero density estimates for the Riemann zeta function and estimates coming up in connection with Piatetski-Shapiro primes.

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MSC:

11P32 Goldbach-type theorems; other additive questions involving primes
11P05 Waring's problem and variants
11N36 Applications of sieve methods

Keywords:

additive theory of prime numbers; fractional powers; sieve methods

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