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Sharp effective Nullstellensatz. (English) Zbl 0682.14001

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Let K be any field and n, $d_1, ..., d_k$ be natural numbers. Let $N(n, \underline{d}) = \min\{s| \quad for$ any homogeneous polynomials $\phi_1, ..., \phi_k \in K[X_0, ..., X_n]$ such that $\deg(\phi_i) = d_i$, we have $(rad(I))^s \subset I$ where $I = (\phi_1, ..., \phi_k)\}$.

Main theorem: If we assume $d_1 \geq d_2 \geq ... \geq d_k$ and $d_i \neq 2$ (for *all* $\quad i$), then N(n,d)= $d_1...d_k$ if $k \leq n$, N(n,d)= $d_1...d_{n-1} \cdot d_k$ if $k > n > 1$, N(n,d)= $d_1 + d_k - 1$ if $k > n = 1$.

Corollary. Let $f_1, ..., f_k, h \in K[X_1, ..., X_n]$ and assume that h vanishes on all common zeros of $f_1, ..., f_k$ in the algebraic closure of K. Let $d_i = \deg(f_i) \neq 2$ (for *all* $\quad i$). Then one can find $g_1, ..., g_k \in K[X_1, ..., X_n]$ and a natural number s such that $\sum g_i f_i = h^s$, $s \leq N(n, \underline{d})$, $\deg(g_i f_i) \leq (1 + \deg(h)) \cdot N(n, \underline{d})$. In particular, if $h = 1$ (so that $f_1, ..., f_k$ have no common zeros), we can choose g_i such that $\sum g_i f_i = 1$ and $\deg(g_i f_i) \leq N(n, \underline{d})$.

This estimate is the best possible. The condition $d_i \neq 2$ is technical, and the author expects that it is not necessary. The proof uses elementary methods of algebraic geometry including local cohomology, and works in all characteristics.

Reviewer: H.Matsumura

MSC:

14A05	Relevant commutative algebra
13F20	Polynomial rings and ideals; rings of integer-valued polynomials

Keywords:

Nullstellensatz; local cohomology

Full Text: DOI