Fujii, Masatoshi; **Matsumoto, Akemi**; **Tominaga, Masaru**
Simultaneous extensions of Diaz-Metcalf and Buzano inequalities. (English) Zbl 06820444
Nihonkai Math. J. 27, No. 1-2, 17-27 (2016)

Summary: We give a simultaneous extension of Diaz-Metcalf and Buzano inequalities: Let $z_1, \dots, z_m$ be nonzero vectors in a Hilbert space $\mathscr{H}$. Suppose that $x_1, \dots, x_n \in \mathscr{H}$ satisfy that for each $j = 1, \dots, m$ there exists a constant r_j such that $0 \le r_j \le \frac{\operatorname{Re}\langle x_i, z_j\rangle}{\|x_i\|}$ for $i = 1, \dots, n$. If $y_1, y_2 \in \mathscr{H}$ satisfy $\langle y_k, z_j\rangle = 0$ for $k = 1, 2$ and $j = 1, \dots, m$, then

$$\left|\left\langle \sum x_i, y_1\right\rangle \left\langle \sum x_i, y_2\right\rangle\right| + \left(\sum \frac{r_j^2}{c_j}\right)\left(\sum \|x_i\|\right)^2 \mathcal{B}\left(y_1 y_2\right) \le \mathcal{B}\left(y_1 y_2\right)\left\|\sum x_i\right\|^2,$$

where $\mathcal{B}(y_1, y_2) := \frac{1}{2}(\|y_1\|\|y_2\| + |\langle y_1, y_2\rangle|)$ and $c_j = \sum_h |\langle z_h, z_j\rangle|$ for $j = 1, \dots, m$.

MSC:

47A63 Linear operator inequalities

Keywords:

Diaz-Metcalf inequality; Selberg inequality; Buzano inequality; Furuta inequality; Heinz-Kato-Furuta inequality; chaotic order

Full Text: Euclid