zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Bonotto, E. M.; Federson, M.; Santos, F. L.

Dichotomies for generalized ordinary differential equations and applications. (English)

Zbl 1396.34034
J. Differ. Equations 264, No. 5, 3131-3173 (2018).

The paper develops a basic theory of ordinary and exponential dichotomies for generalized linear differential equations of the form

$$x(t) = x(t_0) + \int_{t_0}^{t} \mathrm{d}[A(s)]x(s),$$

where $A : J \subset \mathbb{R} \to \mathcal{L}(X)$ is a function with locally bounded variation, and solutions taking values in a Banach space X.

The results are analogues of the classical theory presented in *W. A. Coppel*'s book [Dichotomies in stability theory. Springer, Cham (1978; Zbl 0376.34001)], and include the relation between the exponential dichotomy and the existence of a unique bounded solution to the inhomogeneous equation

$$x(t) = x(t_0) + \int_{t_0}^{t} \mathrm{d}[A(s)]x(s) + f(t) - f(t_0).$$

In the second part of the paper, the authors show that their results are also applicable to measure differential equations and impulsive differential equations, which represent a special case of generalized ODEs. Reviewer's remarks:

1) A correct reference for Theorem 4.1 is [*Š. Schwabik*, Math. Bohem. 125, No. 4, 431–454 (2000; Zbl 0974.34057)]; the finite-dimensional proof from [*Š. Schwabik*, Generalized ordinary differential equations. Singapore: World Scientific (1992; Zbl 0781.34003), Theorem 6.17] no longer works in the context of Banach spaces.

2) Condition (A3) on page 3158 might be simplified, since $I + \lim_{r\to t+} \int_t^r G(s)\,\mathrm{d}u(s) = I + G(t)\Delta^+ u(t)$.

Reviewer: Antonín Slavík (Praha)

MSC:

34D09 Dichotomy, trichotomy of solutions to ordinary differential equations
45A05 Linear integral equations
34G10 Linear differential equations in abstract spaces
34A36 Discontinuous ordinary differential equations

Cited in **3** Documents

Keywords:
generalized ordinary differential equation; dichotomy; exponential dichotomy; Kurzweil integral

Full Text: DOI

References:

[1] Afonso, S. M.; Bonotto, E. M.; Federson, M., On exponential stability of functional differential equations with variable impulsive perturbations, Differential Integral Equations, 27, 721-742, (2014) · Zbl 1340.34303
[2] Afonso, S. M.; Bonotto, E. M.; Federson, M.; Gimenes, L. P., Boundedness of solutions for functional differential equations with variable impulses via generalized ordinary differential equations, Math. Nachr., 285, 545-561, (2012) · Zbl 1252.34076
[3] Afonso, S. M.; Bonotto, E. M.; Federson, M.; Schwabik, Š., Discontinuous local semiflows for kurzweil equations leading to Lasalle's invariance principle for differential systems with impulses at variable times, J. Differential Equations, 250, 2936-3001, (2011) · Zbl 1213.34019
[4] Bainov, D. D.; Kostadinov, S. I.; van Minh, Nguyen, Dichotomies and integral manifolds of impulsive differential equations, (1994), Oxford Graphics Printers Singapore
[5] Bartle, R. G., A modern theory of integration, Graduate Studies in Mathematics, vol. 32, (2000), AMS

[6] Coffman, C. V.; Schäfer, J. J., Dichotomies for linear difference equations, Math. Ann., 172, 139-166, (1967) · Zbl 0189.40303

[7] Coppel, W. A., Dichotomies in stability theory, Lecture Notes in Mathematics, (1978), Springer-Verlag Berlin, Heidelberg, New York · Zbl 0376.34001

[8] Daleckii, Ju. L.; Krein, M. G., Stability of solutions of differential equations in Banach space, Translations of Mathematical Monographs, vol. 43, (1974), American Mathematical Society Providence, Rhode Island

[9] Federson, M., Substitution formulas for the kurzweil and Henstock vector integrals, Math. Bohem., 127, 1, 15-26, (2002) · Zbl 1002.28012

[10] Federson, M.; Mesquita, J. G.; Slavík, A., Measure functional differential equations and functional dynamic equations on time scales, J. Differential Equations, 252, 3816-3847, (2012) · Zbl 1239.34076

[11] Federson, M.; Schwabik, Š., Generalized ODE approach to impulsive retarded differential equations, Differential Integral Equations, 19, 11, 1201-1234, (2006) · Zbl 1212.34251

[12] Federson, M.; Táboas, P. Z., Topological dynamics of retarded functional differential equations, J. Diff. Equations, 195, 2, 313-331, (2003) · Zbl 1054.34102

[13] Henry, D., Geometric theory of semilinear parabolic equations, Lecture Notes in Mathematics, vol. 840, (1980), Springer-Verlag Berlin-Heidelberg-New York

[14] Henstock, R., Lectures on the theory of integration, (1988), World Scientific Singapore · Zbl 0668.28001

[15] Hönig, C. S., Volterra Stieltjes-integral equations, (1975), North-Holland Publ. Comp Amsterdam

[16] Imaz, C.; Vorel, Z., Generalized ordinary differential equations in Banach spaces and applications to functional equations, Bol. Soc. Mat. Mexicana, 11, 47-59, (1966) · Zbl 0178.44203

[17] Kurzweil, J., Generalized ordinary differential equations and continuous dependence on a parameter, Czechoslovak Math. J., 7, 82, 418-448, (1957) · Zbl 0090.30002

[18] Kurzweil, J., Generalized ordinary differential equations, Czechoslovak Math. J., 8, 83, 360-388, (1958) · Zbl 0094.05804

[19] Lin, Z. S.; Lin, Y.-X., Linear systems exponential dichotomy and structure of sets of hyperbolic points, (1999), Word Scientific Publishing Co. Pte. Ltd. Singapore

[20] Naralenkov, K. M., On integration by parts for Stieltjes-type integrals of Banach space-valued functions, Real Anal. Exchange, 30, 1, 235-260, (2004/2005) · Zbl 1069.28008

[21] Oliva, F.; Vorel, Z., Functional equations and generalized ordinary differential equations, Bol. Soc. Mat. Mexicana, 11, 40-46, (1966) · Zbl 0178.44204

[22] Palmer, K. J., A generalization of Hartman's linearization theorem, J. Math. Anal. Appl., 41, 753-758, (1973) · Zbl 0272.34056

[23] Palmer, K. J., Exponential dichotomies, the shadowing lemma and transversal homoclinic points, (Kircraber, U.; Walther, H. O., Dynamics Reported, vol. 1, (1988), John Wiley and Sons Chichester-New York-Brisbane-Toronto-Singapore), 265-306 · Zbl 0676.58025

[24] Sakamoto, K., Estimates on the strength of exponential dichotomies and application to integral manifolds, J. Differential Equations, 107, 259-279, (1994) · Zbl 0811.34007

[25] Schwabik, Š., Abstract Perron-Stieltjes integral, Math. Bohem., 121, 425-447, (1996) · Zbl 0879.28021

[26] Schwabik, Š., Generalized ordinary differential equations, Ser. Real Anal., vol. 5, (1992), World Scientific Singapore · Zbl 0781.34003

[27] Schwabik, Š., Linear Stieltjes integral equations in Banach spaces, Math. Bohem., 124, 433-457, (1999) · Zbl 0937.34047