Ashraf, Mohd.; **Quadri, Murtaza A.**

On commutativity of associative rings with constraints involving a subset. (English)

Zbl 0683.16025
Rad. Mat. 5, No. 1, 141-149 (1989).

Let R be a ring, N its set of nilpotent elements, and A a non-empty subset of R. Define R to have property (II-A) if for each non-central $x \in R$, there exists a polynomial p(t)$\in \mathbb{Z}[t]$ such that $x - x^2 p(x) \in A$; and let (*) be the property that for each $x, y \in R$, there exist integers $m = m(x, y) > 1$ and $n = n(x, y) \geq 1$ for which $[x, xy - y^m x^n] = 0$. Theorem 1 asserts that R is commutative iff R has property (*) and also satisfies (II-A) for some commutative $A \subseteq N$. Theorem 2 establishes that each of the following is equivalent to commutativity in left s-unital rings: (i) R has property (*) and there exists $A \subseteq N$ for which R has property (II-A); (ii) for some fixed integer $n \geq 1$, R has the property that for each $y \in R$, there exists $m = m(y) > 1$ such that $[x, xy - y^m x^n] = [x, xy^m - y^{m^2} x^n] = 0$ for all $x \in R$. The authors give examples which preclude certain extensions of these results; and they conjecture that property (*) implies commutativity in rings with 1.

Reviewer: H.E.Bell

MSC:

16U70 Center, normalizer (invariant elements) (associative rings and algebras)
16Rxx Rings with polynomial identity

Keywords:

polynomial constraints; commutator constraints; nilpotent elements; commutativity; left s-unital rings