

Gan, Wee Liang; Watterlond, John

A representation stability theorem for VI-modules. (English) Zbl 06839337
Algebr. Represent. Theory 21, No. 1, 47-60 (2018).

Summary: Let VI be the category whose objects are the finite dimensional vector spaces over a finite field of order q and whose morphisms are the injective linear maps. A VI -module over a ring is a functor from the category VI to the category of modules over the ring. A VI -module gives rise to a sequence of representations of the finite general linear groups. We prove that the sequence obtained from any finitely generated VI -module over an algebraically closed field of characteristic zero is representation stable – in particular, the multiplicities which appear in the irreducible decompositions eventually stabilize. We deduce as a consequence that the dimension of the representations in the sequence $\{V_n\}$ obtained from a finitely generated VI -module V over a field of characteristic zero is eventually a polynomial in q^n . Our results are analogs of corresponding results on representation stability and polynomial growth of dimension for FI -modules (which give rise to sequences of representations of the symmetric groups) proved by Church, Ellenberg, and Farb.

MSC:

20C33 Representations of finite groups of Lie type

Cited in 1 Review

Keywords:

representation stability; multiplicity stability; finite general linear groups; VI-modules

Full Text: [DOI](#) [arXiv](#)

References:

- [1] Church, T; Ellenberg, J; Farb, B, FI-modules and stability for representations of symmetric groups, *Duke Math. J.*, 164, 1833-1910, (2015) · [Zbl 1339.55004](#)
- [2] Church, T; Ellenberg, J; Farb, B; Nagpal, R, FI-modules over Noetherian rings, *Geom. Top.*, 18-5, 2951-2984, (2014) · [Zbl 1344.20016](#)
- [3] Church, T; Farb, B, Representation theory and homological stability, *Adv. Math.*, 245, 250-314, (2013) · [Zbl 1300.20051](#)
- [4] Farb, B, Representation stability, *Proc. Int. Congress Math. Seoul, II*, 1173-1196, (2014) · [Zbl 1373.55006](#)
- [5] Gan, WL; Li, L, Noetherian property of infinite EI categories, *New York J. Math.*, 21, 369-382, (2015) · [Zbl 1327.18003](#)
- [6] Gan, W.L., Li, L.: On central stability, to appear in *Bulletin London Mathematical Society*, arXiv:1504.07675 · [Zbl 1373.55006](#)
- [7] Green, JA, The characters of the finite general linear groups, *Trans. Amer. Math. Soc.*, 80, 402-447, (1955) · [Zbl 0068.25605](#)
- [8] MacDonald, I.G.: *Symmetric functions and Hall polynomials*, 2nd edn. Oxford Math. Monographs, Oxford Scientific Publications, The Clarendon Press, Oxford University Press, New York (1995) · [Zbl 0824.05059](#)
- [9] Putman, A., Sam, S.: Representation stability and finite linear groups, to appear in *Duke Mathematical Journal*, arXiv:1408.3694 · [Zbl 1408.18003](#)
- [10] Sam, S., Snowden, A.: Gröbner methods for representations of combinatorial categories, to appear in *Journal of the American Mathematical Society*, arXiv:1409.1670
- [11] Springer, T; Zelevinsky, A, Characters of $\text{GL}(n, \mathbb{F}_q)$ and Hopf algebras, *J. London Math. Soc.* (2), 30, 27-43, (1984) · [Zbl 0551.20022](#)
- [12] Zelevinsky, A.: *Representations of finite classical groups. A Hopf algebra approach* *Lecture Notes in Mathematics*, vol. 869. Springer-Verlag, Berlin-New York (1981) · [Zbl 0465.20009](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.