The paper under review studies some profinite invariants of 3-manifolds, in particular of graph manifolds. Recall that a profinite group is an inverse limit of a system of finite groups, while the profinite completion of a group $G$ is the profinite group $\hat{G}$ defined as the inverse limit of all finite quotient groups of $G$. On the other hand, a graph manifold is an irreducible 3-manifold whose JSJ decomposition consists only of Seifert-fibred pieces.

Now, let $G$ and $H$ be two groups with isomorphic profinite completions. A property $\mathcal{P}$ is called a profinite invariant whenever the group $G$ has $\mathcal{P}$ if and only if $H$ has $\mathcal{P}$.

In this work the author considers only profinite invariants of fundamental groups of compact orientable graph manifolds, and hence one can call them profinite invariants of 3-manifolds. Examples of such invariants are e.g. the geometry of a 3-manifold and the triviality of the JSJ decomposition, as shown in [H. Wilton and P. Zalesskii, Geom. Topol. 21, No. 1, 345–384 (2017; Zbl 1361.57023)].

Obviously, the strongest profinite invariant of a group $G$ is the isomorphism type of the group itself, namely whenever one has that $G \simeq H$ implies $\hat{G} \simeq \hat{H}$. In such a case one says that the group is profinitely rigid. The study of profinitely rigid 3-manifold groups is very much recent and there are only few examples of such manifolds, like for instance most closed Seifert fibred spaces, the figure-eight knot complement, and once-punctured torus bundles (by results of Bridson and others).

In the paper under review the author finds some criteria that determine when two graph manifold groups have isomorphic profinite completion. In particular, the main result states that if two closed orientable graph manifolds have isomorphic completions of their fundamental groups then the graphs of their JSJ decompositions are isomorphic in very precise terms. For instance, given a closed oriented graph manifold $M$, there are only a finite number of closed oriented graph manifolds $N$ with $\pi_1(N) \simeq \pi_1(M)$.

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