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A weighted gradient inequality. (English) Zbl 0686.26004

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Hardy's averaging operator and its dual given by $(P_1f)(x) = \frac{1}{x}\int_0^x f(t)dt$ and $(Q_1f)(x) = \int_x^\infty f(t)\frac{dt}{t}$ satisfy differential equations whose natural analogues for operators on functions on R^n are $x.\nabla(P_nf)(x)+(P_nf)(x) = f(x)$ and $x.\nabla(Q_nf)(x) + f(x) = 0$, respectively. The solutions $(P_nf)(x) = \int_0^1 f(\lambda x)d\lambda$ and $(Q_nf)(x) = \int_1^\infty f(\lambda x)\frac{d\lambda}{\lambda}$ are not dual operators for $n > 1$, but it is shown that weighted norm inequalities for Q_n yield other ones for P_n. The author then concentrates on the inequality

$$(1) \quad (\int_{R^n} |(Q_nf)(x)|^q v(x)dx)^{1/q} \le c(\int_{R^n} |f(x)|^p u(x)dx)^{1/p},$$

where v, u are weights, i.e. non-negative measurable functions on R^n, and $1 \le p < \infty, \quad 0 < q < \infty$. The change to polar coordinates enables to utilise the one-dimensional Hardy inequalities to obtain conditions equivalent to (1) in the cases $1 < p = q < \infty$ or $0 < q < p < \infty$ with $p > 1$ or $1 \le p < q < \infty$ (particularly, in the last case, if $n > 1$ and u is locally integrable, then (1) holds for every f if and only if $v = 0$). As a consequence, conditions are derived for the inequality

$$(\int_{R^n} |g(x)|^q v(x)dx)^{1/q} \le c(\int_{R^n} |x.\nabla g(x)|^p u(x)dx)^{1/p}$$

to hold for every $g \in C_0^\infty(R^n)$.

Reviewer: J.Rákosník

MSC:

26D10	Inequalities involving derivatives and differential and integral operators	Cited in **2** Reviews Cited in **11** Documents

Keywords:

gradient inequality; n-dimensional analogue; Hardy's averaging operator; weighted norm inequalities

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References:

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