Read, C. J.

The invariant subspace problem on some Banach spaces with separable dual. (English)
Zbl 0686.47010
Proc. Lond. Math. Soc., III. Ser. 58, No. 3, 583-607 (1989).

The existence of a cyclic operator T (i.e., an operator without nontrivial closed invariant subspaces) is demonstrated on Banach spaces X of the following type

$$(a) \quad X = c_0 \oplus W, \quad (b) \quad X = J_\infty \oplus W,$$

where W is an arbitrary separable Banach space and $J_\infty = \ell_2 \oplus \otimes_{i=1}^{\infty} J_i$, the ℓ_2-sum of an infinite sequence of copies of the James space

$$J_i = J = \{a \in c_0| \quad \|a\| = \sup\{(\sum_{i=1}^{n-1} |a_{p_{i+1}} - a_{p_i}|^2)^{1/2}| \quad n \in \mathbb{N}, \quad p_1 < p_2 < ... < p_n\} < \infty\};$$

observe that c_0 has a separable dual and J_∞ has a separable bidual.

The required operator T is constructed as a nuclear perturbation of the sum of weighted shifts on c_0, resp. ℓ_2 and $\otimes_{i=1}^{\infty} J_i$, first on a dense subspace F of X, then, by continuation, on X. The proof is rather technical, as seems unavoidable for this type of results, but it is well-presented. Since the existence of a (hyper) cyclic operator T on $\ell_1 \oplus W$, W separable, has already been shown [*C. J. Read*, Isr. J. Math. 63, 1–40 (1988; Zbl 0782.47002)], the author concludes by stating "that we cannot go much further until and unless we solve the invariant subspace problem for a reflexive Banach space".

Reviewer: G.P.A.Thijsse

MSC:

47A15 Invariant subspaces of linear operators
46B25 Classical Banach spaces in the general theory

Keywords:

cyclic operator; ℓ_2-sum of an infinite sequence of copies of the James space; nuclear perturbation of the sum of weighted shifts

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