Fleming, W. H.; **Souganidis, P. E.**
On the existence of value functions of two-player, zero-sum stochastic differential games.
(English) Zbl 0686.90049
Indiana Univ. Math. J. 38, No. 2, 293-314 (1989).

The goal of this paper is to extend the deterministic, finite horizon, theory of differential games to the stochastic case. The dynamics are given by the Ito equation with controlled diffusion $d\xi(\tau) = f(\tau,\xi,\eta,\zeta)d\tau + \sigma(\tau,\xi,\eta,\zeta)$ dw(τ) on $(\tau$,T], with $\xi(t) = x$. Player η is the maximizer and player ζ is the minimizer of the payoff

$$P_{t,x}(\eta,\zeta) = E_{t,x}\{\int_t^T h(r,\xi(r),\eta(r),\zeta(r))dr \quad + \quad g(\xi(T))\}.$$

One of the difficulties encountered in the stochastic case that is not present in the deterministic version is that the strategies for the players must be defined to be nonanticipating with respect to controls and with respect to the process w$(\cdot)$. This results in a very technical measurability question in proving that the upper and lower value function $V^{\pm}$ defined using Elliott and Kalton's formulation of differential games, actually are viscosity solutions of the upper and lower Isaacs equation, respectively. Of course this is now a second order, parabolic, possibly degenerate if the diffusion degenerates, pde. The authors are able to circumvent this problem by introducing "restrictive strategies". Let $V_1^{\pm}$ denote the upper and lower value functions using the restrictive strategies and let v, respectively u, be the unique viscosity solutions of the lower, respectively upper Isaacs equations. By developing the dynamic programming principle the authors are able to prove that $V_1^- \leq v$ and $V_1^+ \geq u$. It is then proved that $V^- \geq v$ and $V^+ \leq u$. Since it is also true that $V^- \leq V_1^-$ and $V_1^+ \geq V^+$, it then follows that equality holds. The principle result then is that the upper and lower value functions defined without the use of the restrictive strategies will be the unique viscosity solutions of the corresponding Isaacs equation.

Reviewer: E.Barron

MSC:

91A23	Differential games (aspects of game theory)
91A15	Stochastic games, stochastic differential games
91A05	2-person games
91A20	Multistage and repeated games

Keywords:

Ito equation; controlled diffusion; viscosity solutions; upper and lower Isaacs equation; restrictive strategies

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