Let \( \{\phi_0, \phi_1, \ldots\} \) and \( \{\psi_0, \psi_1, \ldots\} \) be sets of classical orthogonal polynomials such as Chebyshev, Legendre, ultraspherical, Jacobi, or Laguerre polynomials. Then the \((N+1) \times (N+1)\) upper-triangular conversion matrix \( A \) is defined by \( (b_k)_{k=0}^N = A (a_j)_{j=0}^N \), where

\[
\sum_{j=0}^N a_j \phi_j(x) = \sum_{k=0}^N b_k \psi_k(x).
\]

In this interesting paper, the authors describe how to compute \( A (a_j)_{j=0}^N \) in \( O(N \log^2 N) \) arithmetic operations. For this purpose, the conversion matrix \( A \) is decomposed in the form

\[
A = D_1 (T \circ H) D_2,
\]

where \( D_1 \) and \( D_2 \) are diagonal matrices, \( T \) is an upper-triangular Toeplitz matrix, \( H \) is a positive semidefinite Hankel matrix, and \( \circ \) is the (entrywise) Hadamard product. Using a pivoted Cholesky algorithm, the Hankel matrix \( H \) is approximated by a low rank matrix. The Toeplitz matrix-vector products are computed by fast Fourier transform. This algorithm of basis conversion is conceptually simple and requires no precomputation. In this paper, the Legendre-to-Chebyshev basis conversion is mainly discussed. Later, other polynomial basis conversions are investigated. Numerical tests illustrate the high performance of this method.

Reviewer: Manfred Tasche (Rostock)

MSC:

- 65T50 Numerical methods for discrete and fast Fourier transforms
- 42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
- 15B05 Toeplitz, Cauchy, and related matrices

Keywords:

- polynomial basis conversion
- orthogonal polynomials
- conversion matrix
- fast polynomial transforms
- Toeplitz-dot-Hankel matrix
- fast Fourier transform
- Legendre polynomials
- Chebyshev polynomials

Software:

- Clenshaw-Curtis
- FFTW
- ApproxFun
- Julia
- Chebfun
- DLMF
- FastTransforms.jl
- GitHub

Full Text: DOI arXiv

References:


[27] Marsaglia, George; Styan, George P. H., Equalities and inequalities for ranks of matrices, Linear and Multilinear Algebra, 2, 269-292 (1974/75) · Zbl 0297.15003


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