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**Gallai's question and constructions of almost hypotraceable graphs.** (English) Zbl 1387.05072  
*Discrete Appl. Math.* 243, 270-278 (2018).

Summary: Consider a graph  $G$  in which the longest path has order  $|V(G)| - 1$ . We denote the number of vertices  $v$  in  $G$  such that  $G - v$  is non-traceable with  $t_G$ . Gallai asked in 1966 whether, in a connected graph, the intersection of all longest paths is non-empty. *H. Walther* [*J. Comb. Theory* 6, 1–6 (1969; [Zbl 0184.27504](#))] showed that, in general, this is not true. In a graph  $G$  in which the longest path has  $|V(G)| - 1$  vertices, the answer to Gallai's question is positive iff  $t_G \neq 0$ . In this article we study almost hypotraceable graphs, which constitute the extremal case  $t_G = 1$ . We give structural properties of these graphs, establish construction methods for connectivities 1 through 4, show that there exists a cubic 3-connected such graph of order 28, and draw connections to works of *C. Thomassen* [*Discrete Math.* 9, 91–96 (1974; [Zbl 0278.05110](#)); *ibid.* 14, 377–389 (1976; [Zbl 0322.05130](#))] and *L. Gargano et al.* [*ibid.* 285, No. 1–3, 83–95 (2004; [Zbl 1044.05048](#))].

**MSC:**

[05C12](#) Distance in graphs  
[05C38](#) Paths and cycles

Cited in 2 Documents

**Keywords:**

Gallai's problem; traceable; hypotraceable; almost hypotraceable

**Software:**

[GenHypohamiltonian](#)

**Full Text:** [DOI](#)

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