Almost-regular dessins d’enfant on a torus and sphere. (English) Zbl 1396.57004


The paper under review gives a contribution to the realization problem, which consists in classifying which ramification data over the 2-dimensional sphere are realisable. The authors consider ramification data for which – when realisable – the covering space is either the torus or the sphere. A family $T = (T_i)_{i \in \mathbb{N}}$ of ramification data with $r$ branch points is called almost regular of type $[k_1, \ldots, k_r]$ and error at most $\epsilon$, if there exist a tuple $A_j$ of positive integers different from $k_j$ for each $j = 1, \ldots, r$, such that each $T_i$ is of the form $[A_1, k_1^*], \ldots, [A_r, k_r^*]$ with degree tending to infinity, and such that the sum $\Sigma_{j=1}^{r} a \in A_j a \leq \epsilon$.

They then show the following theorem:

Theorem 1.1. Let $T$ be a family of almost-regular genus 1 ramification data of type $[k_1, \ldots, k_r]$, error at most $\epsilon$, where $T_i$ is not one of the exceptional types $(A)-(D)$ (see below) for $i \in \mathbb{N}$. Then all but finitely many members of $T$ are realizable if $\epsilon \leq 6$, or if $[k_1, \ldots, k_r] \in \{[2,2,2,2][3,3,3]\}$ and $\epsilon \leq 10$.

The exceptional types $(A)-(D)$ are:

$(A) [1,3,2^*][2^*][2^*]; (B) [2,4,3^*][3^*][3^*]; (C) [2^*][3,5,4^*][4^*]; (D) [2^*][3^*][5,7,6^*].$

Another result about ramification data of genus 1, in the same spirit, is proved, and for genus zero the authors show:

Theorem 1.4. With the exception of $[2^*][1,3^*][2,2,6^*]$, every family of almost-regular ramification data of genus 0 with $\epsilon \leq 6$ is realisable in infinitely many degrees. Moreover, for families of type $[2,2,2,2]$ or $[3,3,3]$, the same assertion holds with $\epsilon \leq 10$.

The authors use a mix of techniques with a focus on the study of a graph on a surface of genus $g$ which comes from the ramification data, called a “dessin d’enfant”. In the introduction the authors provide useful and detailed information about the state of art of this problem.

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MSC:

57M12 Low-dimensional topology of special (e.g., branched) coverings
57M10 Covering spaces and low-dimensional topology
30F10 Compact Riemann surfaces and uniformization
20B30 Symmetric groups

Keywords:

dessins d’enfants; surface; branched coverings; Riemann-Hurwitz formula; graph

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References:


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