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The Weierstrass mean. I: The periods of $\wp(z|e_1, e_2, e_3)$. (English) Zbl 0689.65005

Numer. Math. 57, No. 8, 737-746 (1990).

One of the most important computations ever made was due to Gauss. From his computations of arithmetic-geometric means he was led to the study of ϑ-functions, modular functions and the transformation theory of elliptic objects, in particular the Landen-transformation. This is translated into Weierstrassian: given $e_1(0) > e_2(0) > e_3(0)$ with $e_1(0) + e_2(0) + e_3(0) = 0$, sequences $\{e_1(n)\}$, $\{e_2(n)\}$, $\{e_3(n)\}$ are defined which converge quadratically and monotonically to 2W, -W, -W, where $W = (\pi/\omega)^2/12$, ω being the real period of $\pi(z|e_1(0), e_2(0), e_3(0))$.

Reviewer: J.Todd

MSC:

65D20 Computation of special functions and constants, construction of tables
65B10 Numerical summation of series
33D10 Basic theta functions (MSC2000)
40A25 Approximation to limiting values (summation of series, etc.)

Keywords:

Weierstrass mean; theta function; Weierstrass P function; quadratic convergence; arithmetic-geometric means; modular functions; elliptic objects; Landen-transformation; period

Full Text: DOI EuDML

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