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Norm inequalities related to Clarkson inequalities. (English) Zbl 1390.15064

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Summary: Let A and B be $n \times n$ matrices. It is shown that if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both $A + B$, $A - B$ are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \leq 2^{p-1} \left(\|A\|_p^p + \|B\|_p^p\right) - \left(2^{p/2} - 2\right) \left|\|A\|_p - \|B\|_p\right|^p,$$

and if $p = 2$, $4 \leq p < \infty$, or $2 < p < 4$, and both A, B are positive semidefinite, then

$$\|A + B\|_p^p + \|A - B\|_p^p \geq 2 \left(\|A\|_p^p + \|B\|_p^p\right) + (2^{1-p/2} - 2^{2-p}) \left|\|A + B\|_p - \|A - B\|_p\right|^p.$$

These inequalities are reversed if $p = 2$, $1 \leq p \leq \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both $A + B$, $A - B$ are positive semidefinite, and if $p = 2$, $1 \leq p \leq \frac{4}{3}$, or $\frac{4}{3} < p < 2$, and both A, B are positive semidefinite, respectively. Commutative (or L_p) versions of these inequalities are also considered.

MSC:

15A45 Miscellaneous inequalities involving matrices
15A60 Norms of matrices, numerical range, applications of functional analysis to matrix theory
47A30 Norms (inequalities, more than one norm, etc.) of linear operators
47B10 Linear operators belonging to operator ideals (nuclear, p-summing, in the Schatten-von Neumann classes, etc.)
46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
15A18 Eigenvalues, singular values, and eigenvectors

Keywords:

Clarkson inequality; Hanner inequality; Schatten p-norm; L_p function; singular value

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