Du, Kun

A note on proof of Gordon's conjecture. (English) Zbl 1397.57038

Bull. Korean Math. Soc. 55, No. 3, 699-715 (2018).

Let M be a 3-manifold. If there is a closed surface S which cuts M into two compression bodies V and W with $S=\partial_+W=\partial_+V$, then we say M has a Heegaard splitting, denoted by $M=V\cup_S W$; and S is called a Heegaard surface of M. If there is an essential disk in each of the two compression bodies, such that the two disks intersect in a single point, then the Heegaard splitting is said to be stabilized and we may find another Heegaard splitting of the 3-manifold with a lower genus. Now suppose M is a reducible 3-manifold such that M=$M_1\#M_2$. There is a standard Heegaard splitting of M= $V\cup_S W$, called the connected sum of $M_1=V_1\cup_{S_1} W_1$ and $M_2=V_2\cup_{S_2} W_2$. Gordon conjectured that $V\cup_S W$ is stabilized if and only if one of $M_1=V_1\cup_{S_1} W_1$ and $M_2=V_2\cup_{S_2} W_2$ are stabilized, and this conjecture has been proven by *D. Bachman* [Geom. Topol. 12, No. 4, 2327–2378 (2008; Zbl 1152.57020)] and *R. Qiu* and *M. Scharlemann* [Adv. Math. 222, No. 6, 2085–2106 (2009; Zbl 1180.57025)]. This paper gives an alternative proof of Gordon's Conjecture by using Qiu's labels and two new labels.

Reviewer: Qiang E (Dalian)

MSC:

57N10	Topology of general 3-manifolds (MSC2010)
57M50	General geometric structures on low-dimensional manifolds
57M27	Invariants of knots and 3-manifolds (MSC2010)

Keywords:

3-manifolds; Heegaard splitting; stabilization; band sum; Gordon's conjecture

Full Text: Link

References:

[1] D. Bachman, Connected sums of unstabilized Heegaard splittings are unstabilized, Geom. Topol. 12 (2008), no. 4, 2327-2378. · Zbl 1152.57020

[2] R. Kirby, Problems in low dimensional manifold theory, in Algebraic and geometric topology (Proc. Sympos. Pure Math., Stanford Univ., Stanford, Calif., 1976), Part 2, 273-312, Proc. Sympos. Pure Math., XXXII, Amer. Math. Soc., Providence, RI, 1978.

[3] T. Kobayashi and R. Qiu, The amalgamation of high distance Heegaard splittings is always efficient, Math. Ann. 341 (2008), no. 3, 707-715. · Zbl 1140.57012

[4] T. Kobayashi, R. Qiu, Y. Rieck, and S. Wang, Separating incompressible surfaces and stabilizations of Heegaard splittings, Math. Proc. Cambridge Philos. Soc. 137 (2004), no. 3, 633-643. · Zbl 1062.57028

[5] M. Lackenby, The Heegaard genus of amalgamated 3-manifolds, Geom. Dedicata 109 (2004), 139-145. · Zbl 1081.57018

[6] R. Qiu, Stabilizations of reducible Heegaard splittings, arXiv: math. GT/0409497.

[7] R. Qiu and M. Scharlemann, A proof of the Gordon conjecture, Adv. Math. 222 (2009), no. 6, 2085-2106. · Zbl 1180.57025

[8] J. Schultens, The classification of Heegaard splittings for (compact orientable surface) ×S1, Proc. London Math. Soc. (3) 67 (1993), no. 2, 425-448. · Zbl 0789.57012

[9] J. Schultens and R. Weidmann, Destabilizing amalgamated Heegaard splittings, in Workshop on Heegaard Splittings, 319-334, Geom. Topol. Monogr., 12, Geom. Topol. Publ., Coventry, 2007. Kun Du School of Mathematics and Statistics Lanzhou University Lanzhou 730000, P. R. China Email address: dukun@lzu.edu.cn · Zbl 1216.57011