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Eigenvalue interlacing for first order differential systems with periodic 2×2 matrix potentials and quasi-periodic boundary conditions. (English) [Zbl 1409.34075](#)

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The paper deals with the self-adjoint Dirac system in the form

$$JY' + QY = \lambda Y, \quad (1)$$

where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} q_1 & q \\ q & q_2 \end{pmatrix}$, the functions q , q_1 and q_2 are real-valued, integrable and π -periodic. It is shown that the eigenvalues of the boundary value problem for equation (1) with the boundary conditions

$$Y(\pi) = \pm R(\theta)Y(\theta),$$

where $R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, coincide with the values of λ , such that the discriminant $\Delta_\theta = \text{Tr}(\mathbb{Y}(\pi)^T R(\theta))$ equals ± 2 . Here $\mathbb{Y}(x)$ is the solution of (1), satisfying the initial condition $\mathbb{Y}(0) = \mathbb{I}$. The authors obtain explicit formulas for the λ -derivative of the discriminant Δ_θ and monotonicity results for the first and the second λ -derivatives. The main results of the paper are several interlacing theorems for the eigenvalues of (1), corresponding to various boundary conditions.

Reviewer: [Natalia Bondarenko \(Saratov\)](#)

MSC:

- [34L40](#) Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)
- [34L15](#) Eigenvalues, estimation of eigenvalues, upper and lower bounds of ordinary differential operators
- [47E05](#) General theory of ordinary differential operators (should also be assigned at least one other classification number in Section 47-XX)

Keywords:

[Dirac system](#); [quasi-periodic eigenvalue problems](#); [interlacing](#)

Full Text: [DOI](#)

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