zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Zamfirescu, Carol T.

Congruent triangles in arrangements of lines. (English) Zbl 1396.52022

Ars Math. Contemp. 14, No. 2, 359-373 (2018).

In the paper under review, the author studies the maximal number of congruent triangles determined by finite arrangements of ℓ lines in $\mathbb{R}^2$. In order to formulate the main results of the paper, we need to recall some notions. An arrangement $\mathcal{A}$ of lines is always a finite family of ℓ lines $L_1, \dots, L_\ell$, which is not a pencil. Denote by $\mathfrak{U}_\ell$ the set of all arrangements of $\ell \geq 3$ lines. For an arrangement $\mathcal{A} \in \mathfrak{U}_\ell$ we can associate a graph $\Gamma_{\mathcal{A}}$: the vertices of $\Gamma_{\mathcal{A}}$ correspond to the intersection points of lines and the edges of $\Gamma_{\mathcal{A}}$ correspond to the line-segments between these vertices. A triangle in $\mathcal{A} \in \mathfrak{U}_\ell$ is the convex hull of the set of intersection points of three non-concurrent pairwise non-parallel lines in $\mathcal{A}$. We denote by $F^{\mathcal{A}}$ the set of all triangles in $\mathcal{A}$. If two triangles $\triangle_1, \triangle_2 \in F^{\mathcal{A}}$ are congruent, then we write $\triangle_1 \sim \triangle_2$. Let $F_1^{\mathcal{A}}, \dots, F_p^{\mathcal{A}}$ be the equivalence classes with respect to $\sim$ such that $\#F_1^{\mathcal{A}} \geq \dots \geq \#F_p^{\mathcal{A}}$. We call a triangle $\triangle \in F^{\mathcal{A}}$ *facial* if it is a face of $\Gamma_{\mathcal{A}}$, i.e., $L \cap \operatorname{int}\triangle = \emptyset$ for all $L \in \mathcal{A}$. Denote by $G^{\mathcal{A}} \subset F^{\mathcal{A}}$ the set of all facial triangles, and we denote by $G_1^{\mathcal{A}}, \dots, G_q^{\mathcal{A}}$ the equivalence classes with respect to $\sim$ such that $\#G_1^{\mathcal{A}} \geq \dots \geq \#G_q^{\mathcal{A}}$. Now we define

$$f(\ell) = \max_{\mathcal{A} \in \mathfrak{U}_\ell} \#F_1^{\mathcal{A}}, \qquad g(\ell) = \max_{\mathcal{A} \in \mathfrak{U}_\ell} \#G_1^{\mathcal{A}}.$$

Additionally, we define $F(\ell)$ ($G(\ell)$, respectively) as the set of all integers u such that there exists an arrangement on ℓ lines having exactly u congruent triangles (congruent facial triangles, respectively). We write $[s..t]$ for the set of all integers u such that $s \leq u \leq t$, put H for F or G, and h is equal to f if $H = G$ or g if $H = G$. Whenever $H(\ell) = [0..h(\ell)]$, we say that $H(\ell)$ is complete.

Theorem 1. One has $f(5) = g(5) = 5$ while $F(5)$ and $G(5)$ are complete.

Thereom 2. One has $f(6) = 8$, $6 \leq g(6) \leq 7$, $F(6)$ is complete, and $[0..6] \subset G(6)$.

Theorem 3. One has $f(7) = 14$, $9 \leq g(7) \leq 11$, $[0..10] \cup \{14\} \subset F(7)$, and $[0..9] \subset G(7)$.

Theorem 4. One has $16 \leq f(8) \leq 22$, $12 \leq g(8) \leq 15$, $[0..16] \setminus \{13\} \subset F(8)$ and $[0..12] \subset G(8)$.

In the last section, the author formulates some natural conjectures, for instance the author predicts that $g(6) = 6$.

Reviewer: Piotr Pokora (Kraków)

MSC:

52C10 Erdős problems and related topics of discrete geometry
52C30 Planar arrangements of lines and pseudolines (aspects of discrete geometry)

Keywords:

arrangement of lines; congruent triangles

Software:

OEIS

Full Text: DOI

References:

2 T. Christ, Database of Combinatorially Different Line Arrangements,http://www.inf. ethz.ch/personal/christt/line_arrangements.php(accessed on 15 June 2016).

3 T. Christ, Discrete Descriptions of Geometric Objects, Ph.D. thesis, ETH Z¨urich, 2011, doi: 10.3929/ethz-a-007018092.

4 P. Erd˝os and G. Purdy, Some extremal problems in geometry III, in: F. Hoffman (ed.), Proceedings of the 6th Southeastern

Conference on Combinatorics, Graph Theory and Computing (Florida Atlantic University, Boca Raton, Florida), Utilitas Mathematica, 1975 pp. 291-308. · Zbl 0328.05018

5 P. Erd˝os and G. Purdy, Some extremal problems in geometry IV, in: F. Hoffman (ed.), Proceedings of the 7th Southeastern Conference on Combinatorics, Graph Theory, and Computing (Louisiana State University, Baton Rouge, Louisiana), Utilitas Mathematica, 1976 pp. 307- 322. · Zbl 0345.52007

6 P. Erd˝os and G. Purdy, Extremal problems in combinatorial geometry, in: R. L. Graham, M. Gr¨otschel and L. Lov´asz (eds.), Handbook of Combinatorics, The MIT Press, Cambridge, Massachusetts, volume 1, 1995 pp. 809-874. · Zbl 0852.52009

7 D. Forge and J. L. Ram´ırez Alfons´ın, Straight line arrangements in the real projective plane, Discrete Comput. Geom.20 (1998), 155-161, doi:10.1007/pl00009373.

14 N. J. A. Sloane, Sequence A006066 (formerly M1334) in The On-Line Encyclopedia of Integer Sequences, published electronically athttps://oeis.org.

15 E. W. Weisstein, Kobon Triangle, from MathWorld—A Wolfram Web Resource,http:// mathworld.wolfram.