Pham Huu Tiep

Irreducible orthogonal decompositions in Lie algebras. (Russian) Zbl 0692.17005

Mat. Sb. 180, No. 10, 1396-1414 (1989).

Let $\mathcal{L}$ be a simple finite-dimensional complex Lie algebra. A decomposition $\mathcal{L} = \oplus_{i=1}^{n} \mathcal{H}_i$ of $\mathcal{L}$ into a direct sum of Cartan subalgebras is called an orthogonal decomposition (OD) if these subalgebras $\mathcal{H}_i$ are mutually orthogonal with respect to the Killing form. Such OD is called irreducible (IOD) if the group $Aut_{OD}(\mathcal{L})$ acts on $\mathcal{L}$ absolutely irreducibly, where $Aut_{OD}(\mathcal{L}) = \{\phi |\ \phi \in Aut(\mathcal{L}) \& \forall i\ \exists j\ \phi\ (\mathcal{H}_i) \subset \mathcal{H}_j\}$. The main result of the paper is the following theorem which gives a solution of the so called weakened problem of Winnie-the-Pooh [see *A. I. Kostrikin*, Group theory, Proc. Conf., Singapore 1987, 171-181 (1989; Zbl 0678.17007)].

Theorem. Lie algebras of types A_{p-2} (p is a prime number, $p \neq 2^d - 1$), C_3, E_7 have no IOD. Lie algebras of types A_{p-1} (p is a prime number), G_2, E_4, E_8, E_6 have IOD, and each of their IOD is standard. The number of Aut($\mathcal{L}$)-conjugate IOD classes is equal to 1 in the first four cases and is equal to 2 in the last case E_8.

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MSC:

17B20 Simple, semisimple, reductive (super)algebras
17B40 Automorphisms, derivations, other operators for Lie algebras and super algebras

Keywords:

simple complex Lie algebras; automorphism group; orthogonal decomposition; problem of Winnie-the-Pooh