Maz'ya, Vladimir G.

Sobolev spaces. Transl. from the Russian by T. O. Shaposhnikova. (English) Zbl 0692.46023

Berlin etc.: Springer-Verlag. xix, 486 p. (1985).

From the author's preface: "In the present monograph we consider various aspects of Sobolev space theory. Attention is paid mainly to the so called imbedding theorems.... We list some questions considered in the book.

1. What are the requirements on the measure μ for the inequality $(\int |u|^q d\mu)^{1/q} \leq C\|u\|_{S_p^\ell}$, where S_p^ℓ is the Sobolev space or its generalization, to hold?

2. What are the minimal assumptions on the domain for the Sobolev imbedding theorem to remain valid? How do these theorems vary under the degeneration of the boundaries? How does the class of admissible domains depend on additional requirements placed upon the behavior of the function near the boundary?

3. How "massive" must a subset e of the domain Ω be in order that "the Friedrichs inequality" $\|u\|_{L_q(\Omega)} \leq C\|\nabla_\ell u\|_{L_p(\Omega)}$ hold for all smooth functions that vanish in a neighborhood of e?"

The book represents the state of art in the modern theory of Sobolev spaces. Based on sophisticated tools conditions for inequalities of the above type have often necessary and sufficient character. The underlying (non-smooth) domains are treated with great care.

The book has 12 chapters. 1: Basic properties of Sobolev spaces (highlights are generalized Sobolev theorems and Hardy inequalities). 2: Inequalities for gradients of functions that vanish on the boundary. 3: On summability of functions in the space $L_1^1(\Omega)$. 4: On the summability of functions in the spaces $L_p^1(\Omega)$. 5. On continuity and boundedness of functions in Sobolev spaces. 6: On functions in the space $\mathrm{BV}(\Omega)$ (generalized derivatives are measures). 7: Certain function spaces, capacities and potentials (Riesz and Bessel potentials, Besov spaces). 8: On summability with respect to an arbitrary measure of functions with fractional derivatives. 9: A variant of capacity. 10: An integral inequality for functions on a cube. 11: Imbedding of the space $\overset{\circ}{L}{}_p^\infty(\Omega)$ into other function spaces. 12: The imbedding $\overset{\circ}{L}{}_p^\ell(\Omega,\nu) \subset W_r^m(\Omega)$.

Reviewer: H.Triebel

MSC:

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

46-02 Research exposition (monographs, survey articles) pertaining to functional analysis

Keywords:

Sobolev space; imbedding theorems; Friedrichs inequality; capacities; Riesz and Bessel potentials; Besov spaces; integral inequality