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On the generalization of the Timoshenko beam model based on the micropolar linear theory: static case. (English) Zbl 1394.74011

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Summary: Three generalizations of the Timoshenko beam model according to the linear theory of micropolar elasticity or its special cases, that is, the couple stress theory or the modified couple stress theory, recently developed in the literature, are investigated and compared. The analysis is carried out in a variational setting, making use of Hamilton's principle. It is shown that both the Timoshenko and the (possibly modified) couple stress models are based on a microstructural kinematics which is governed by kinosthenic (ignorable) terms in the Lagrangian. Despite their difference, all models bring in a beam-plane theory only one microstructural material parameter. Besides, the micropolar model formally reduces to the couple stress model upon introducing the proper constraint on the microstructure kinematics, although the material parameter is generally different. Line loading on the microstructure results in a nonconservative force potential. Finally, the Hamiltonian form of the micropolar beam model is derived and the canonical equations are presented along with their general solution. The latter exhibits a general oscillatory pattern for the microstructure rotation and stress, whose behavior matches the numerical findings.

MSC:

74B05 Classical linear elasticity
74K10 Rods (beams, columns, shafts, arches, rings, etc.)

Full Text: DOI

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