Kuba, Markus; Sulzbach, Henning

On martingale tail sums in affine two-color urn models with multiple drawings. (English)

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Summary: In [“Two-colour balanced affine urn models with multiple drawings. I: Central limit theorems”, Preprint, arXiv:1503.09069; “Two-color balanced affine urn models with multiple drawings. II: Large-index and triangular urns”', Preprint, arXiv:1509.09053], *M. Kuba* and *H. M. Mahmoud* introduced the family of two-color affine balanced Pólya urn schemes with multiple drawings. We show that, in large-index urns (urn index between $\frac{1}{2}$ and 1) and triangular urns, the martingale tail sum for the number of balls of a given color admits both a Gaussian central limit theorem as well as a law of the iterated logarithm. The laws of the iterated logarithm are new, even in the standard model when only one ball is drawn from the urn in each step (except for the classical Pólya urn model). Finally, we prove that the martingale limits exhibit densities (bounded under suitable assumptions) and exponentially decaying tails. Applications are given in the context of node degrees in random linear recursive trees and random circuits.

MSC:

60F15 Strong limit theorems
60C05 Combinatorial probability
60F05 Central limit and other weak theorems
60G42 Martingales with discrete parameter

Keywords:

urn model; martingale central limit theorem; law of the iterated logarithm; large-index urn; triangular urn

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