Varchenko, A. N.

The Euler beta-function, the Vandermonde determinant, the Legendre equation, and critical values of linear functions on a configuration of hyperplanes. I. (Russian) Zbl 0695.33004

Izv. Akad. Nauk SSSR, Ser. Mat. 53, No. 6, 1206-1235 (1989).

Given a set of linear functions $f_1, ..., f_N$ on an n-dimensional affine space V and the configuration $S = \{f_j(v) = 0\}_{j=1}^N$ associated to them, this article constructs a square matrix of hypergeometric type integrals $\int \Delta_J \omega_I$, where $\omega_I = g_{I_0} dg_{I_1} \wedge ... \wedge dg_{I_n}$ with $g_{I_j} = \prod_{i \in I_j} f_i^{\alpha_i}$, Δ_J is a bounded convex component of $V \setminus S$. A formula of the type

$$\det(\int_{\Delta_J} \omega_I) = B(S, \alpha) \prod_{\Delta_J, g_i} c(g_i, \Delta_j)$$

is presented, where $c(g_i(\Delta_J)$ is the maximum of $|g_i|$ on $\bar{\Delta}_J$ and B(S,α) is a constant called beta function of the configuration. In the case $n = 1$, this formula contains in particular the relation between beta and gamma, formulas for the Vandermonde determinant, Legendre's formula for the elliptic functions, and gives new formulas on the determinant of the period matrix of hyperelliptic integrals.

Reviewer: A.Kaneko

MSC:

33C60 Hypergeometric integrals and functions defined by them (E, G, H and I functions)
33B15 Gamma, beta and polygamma functions

Keywords:

period matrix